

Bounded Quantifier Instantiation for Checking Inductive Invariants

TACAS'17
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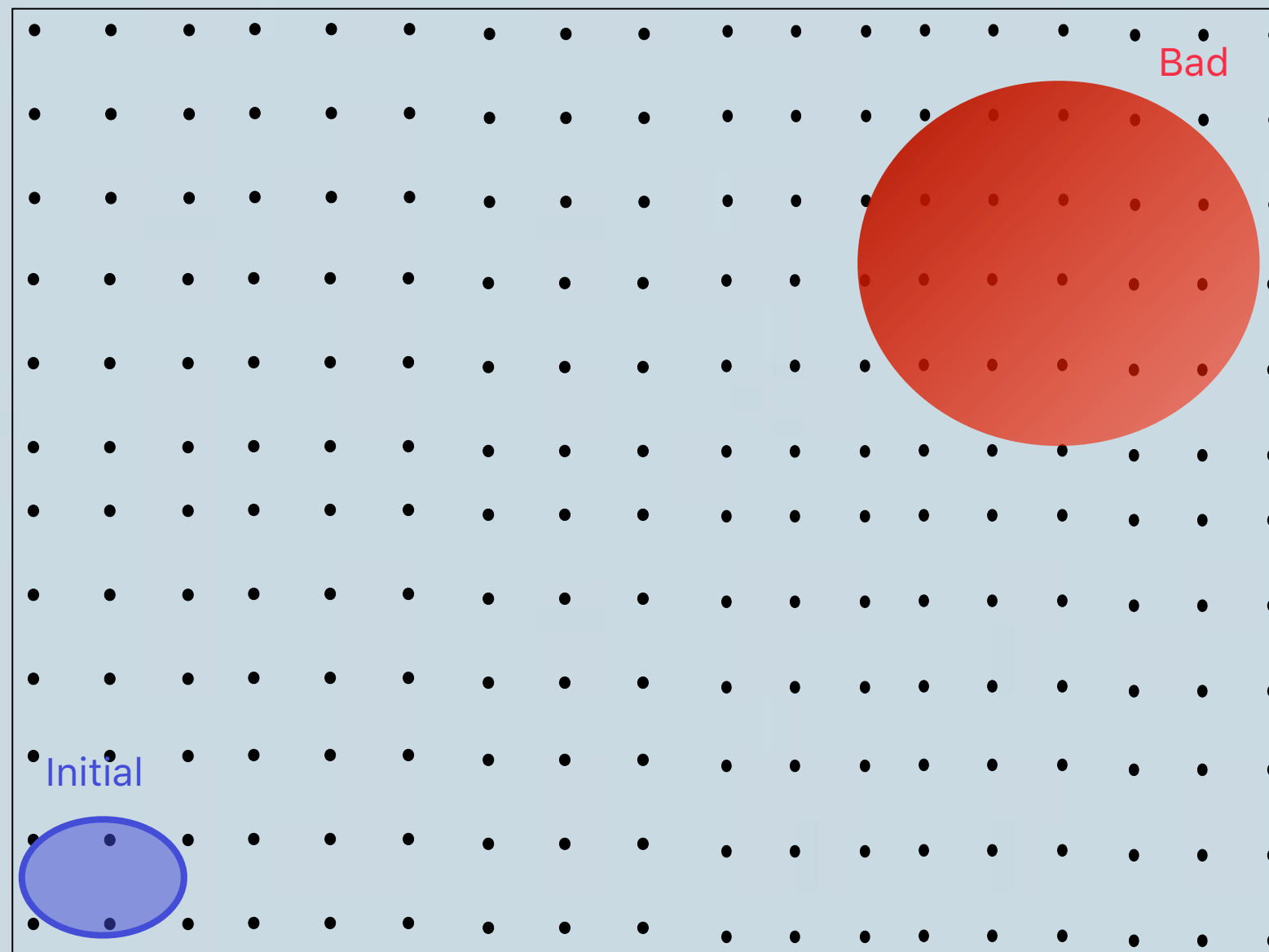
Oded Padon
Tel Aviv University

Neil Immerman
UMass, Amherst

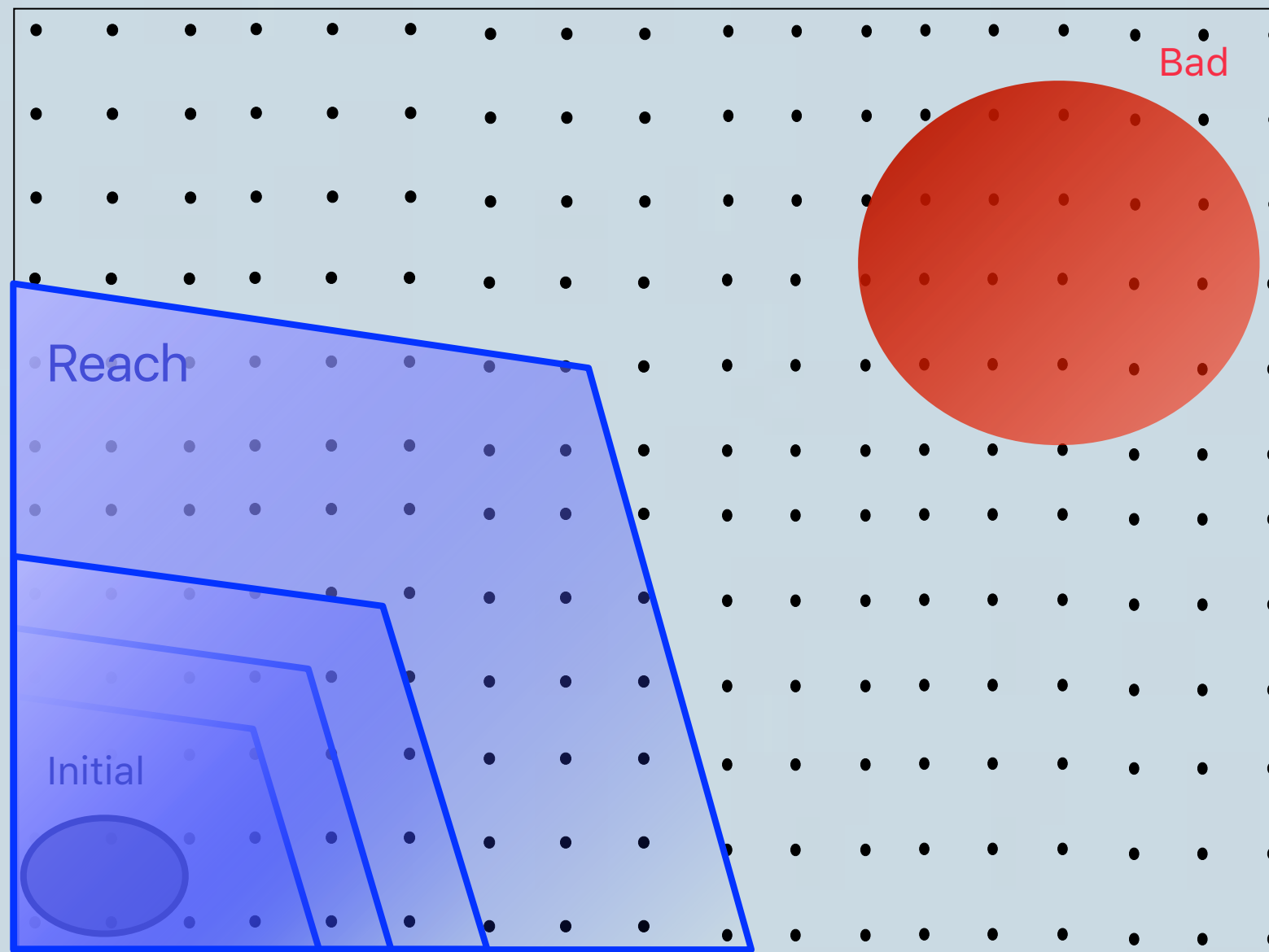
Mooly Sagiv
Tel Aviv University

Sharon Shoham
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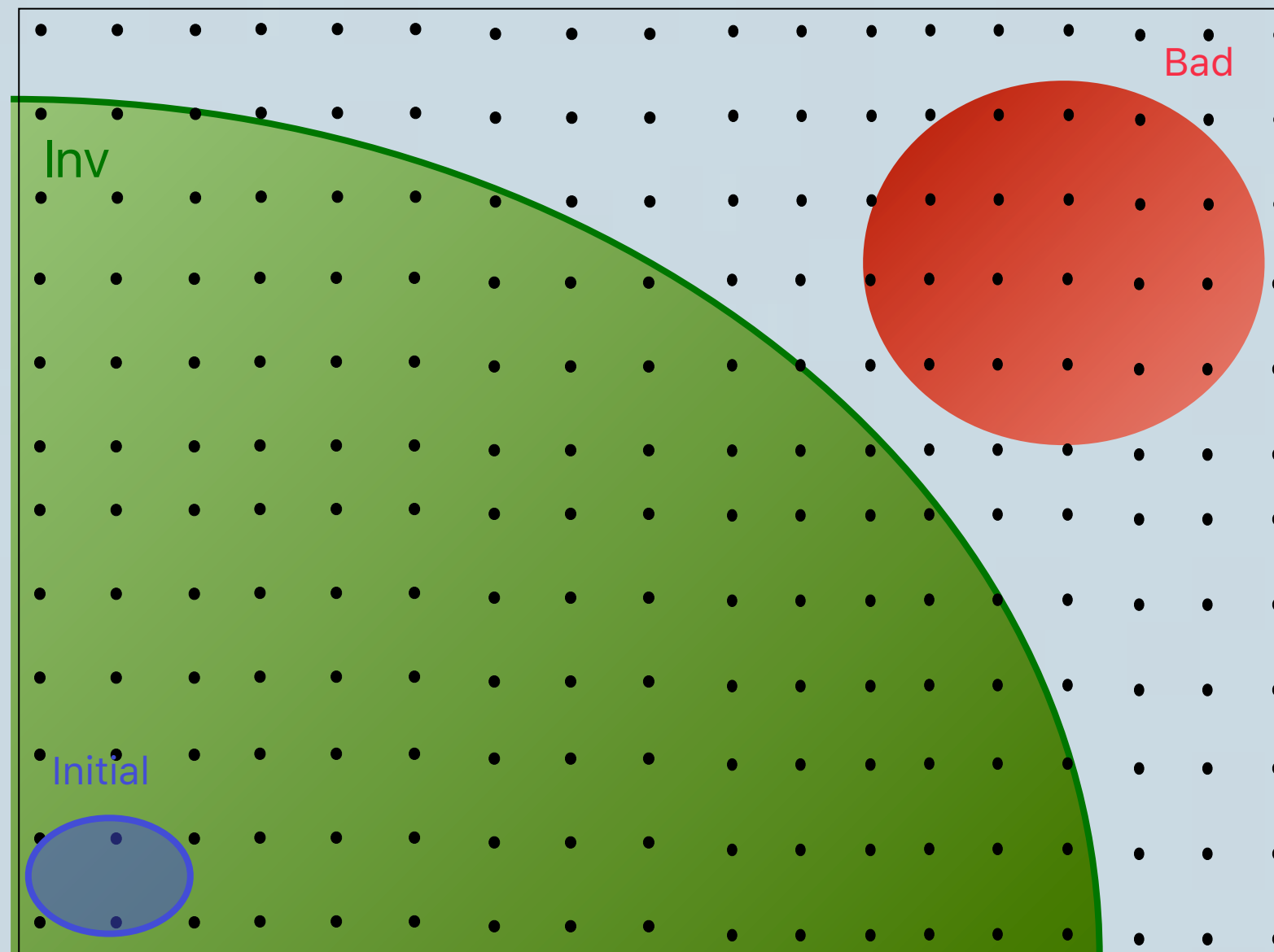
Safety of Infinite-State Systems



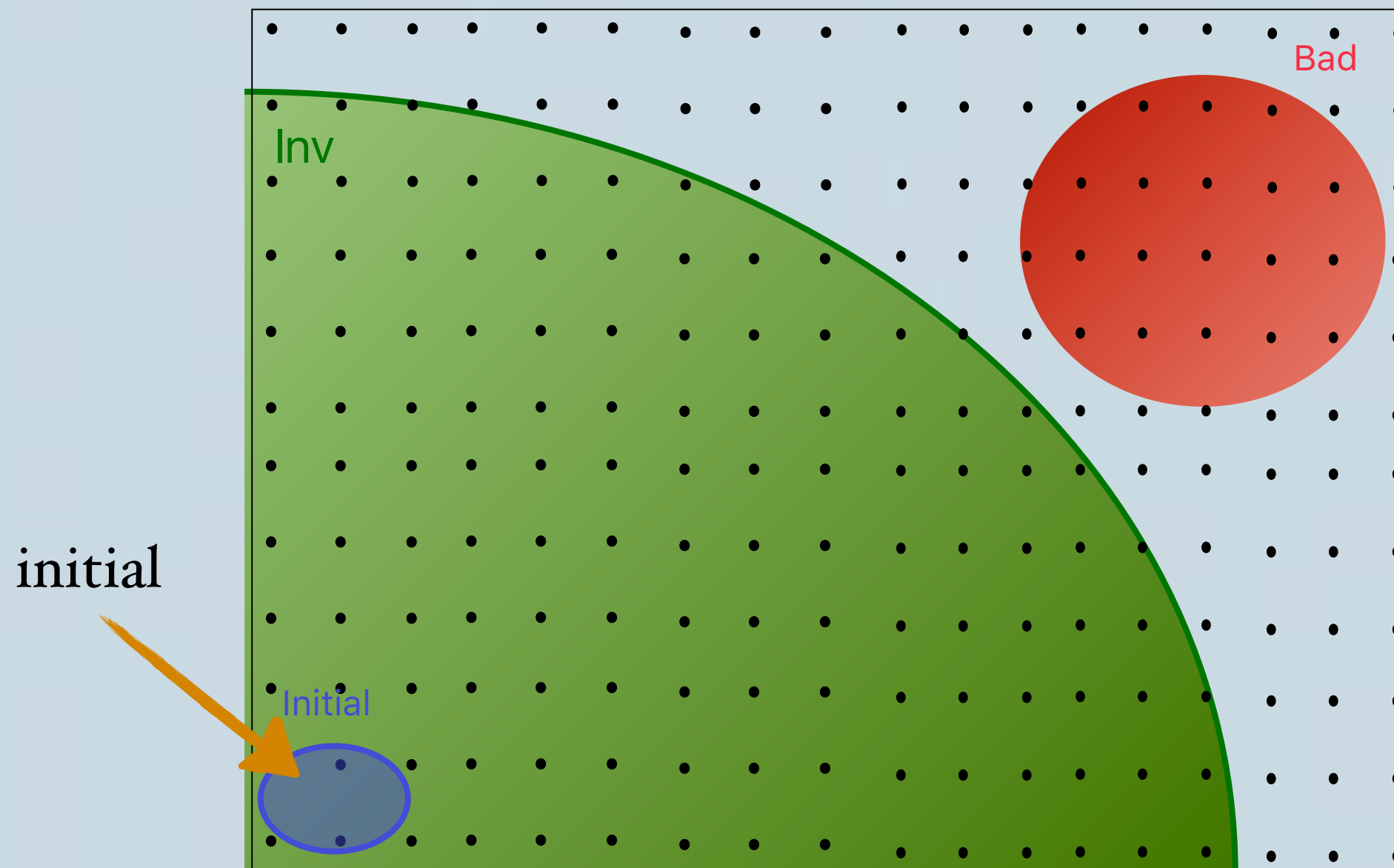
Safety of Infinite-State Systems



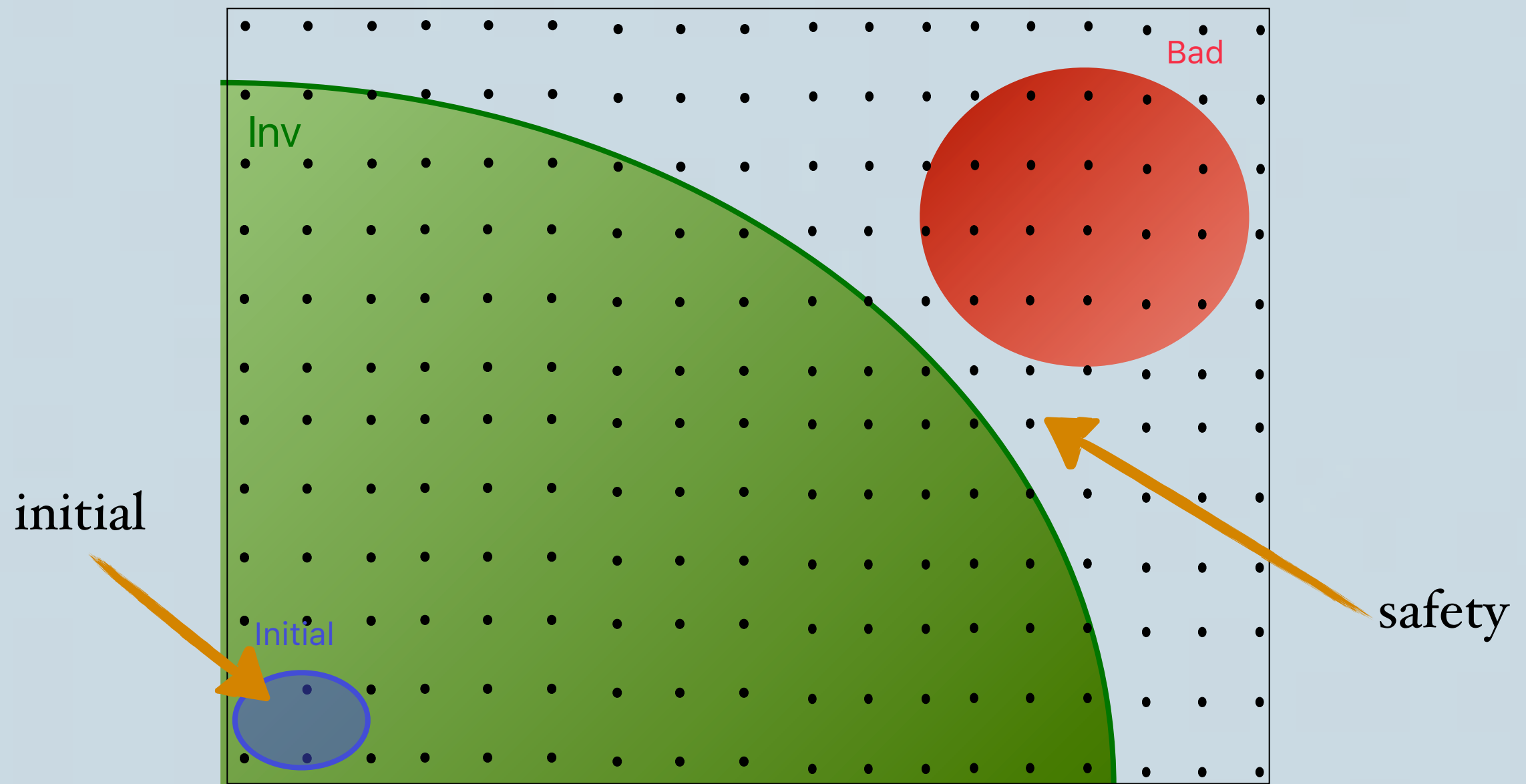
Inductive Invariants



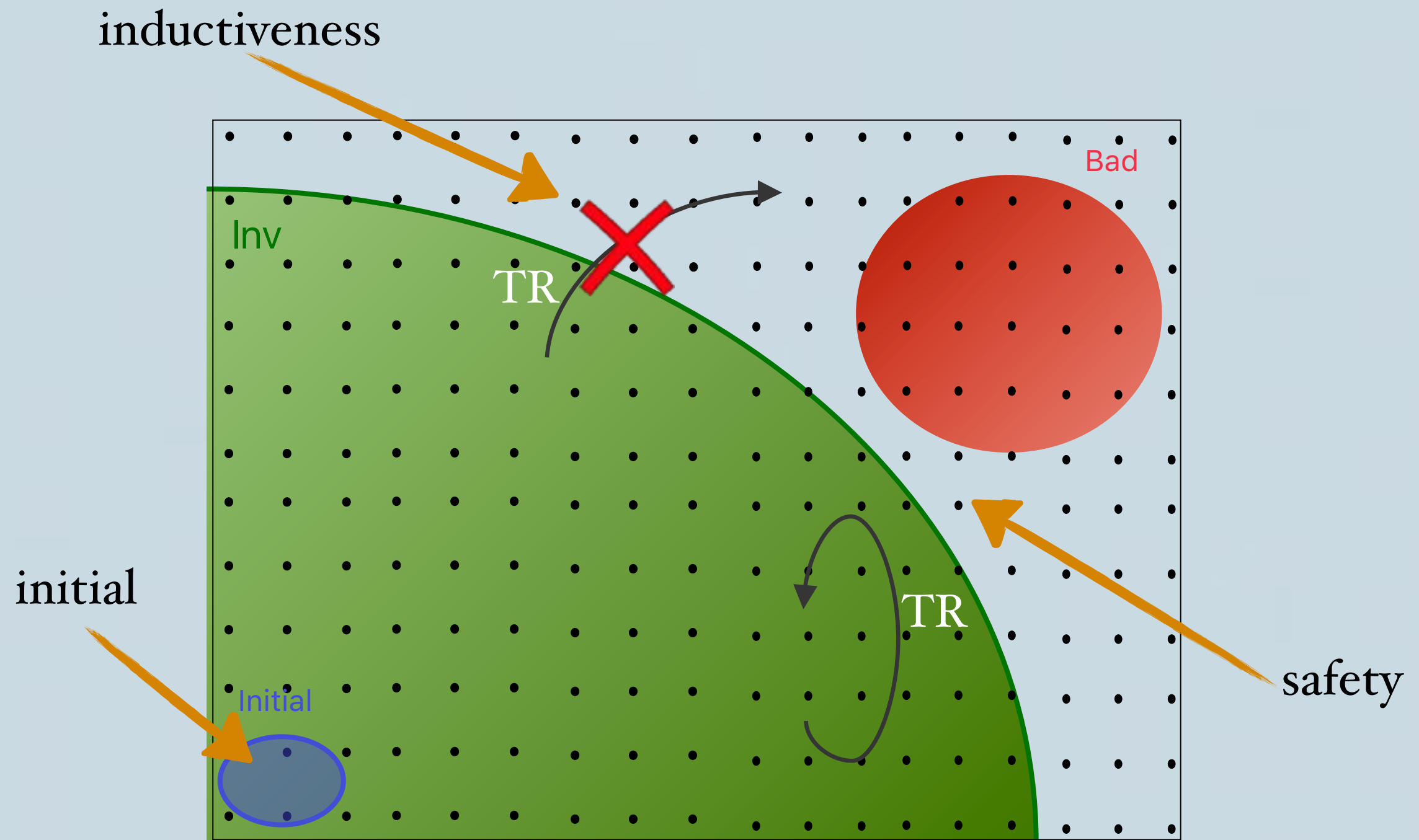
Inductive Invariants



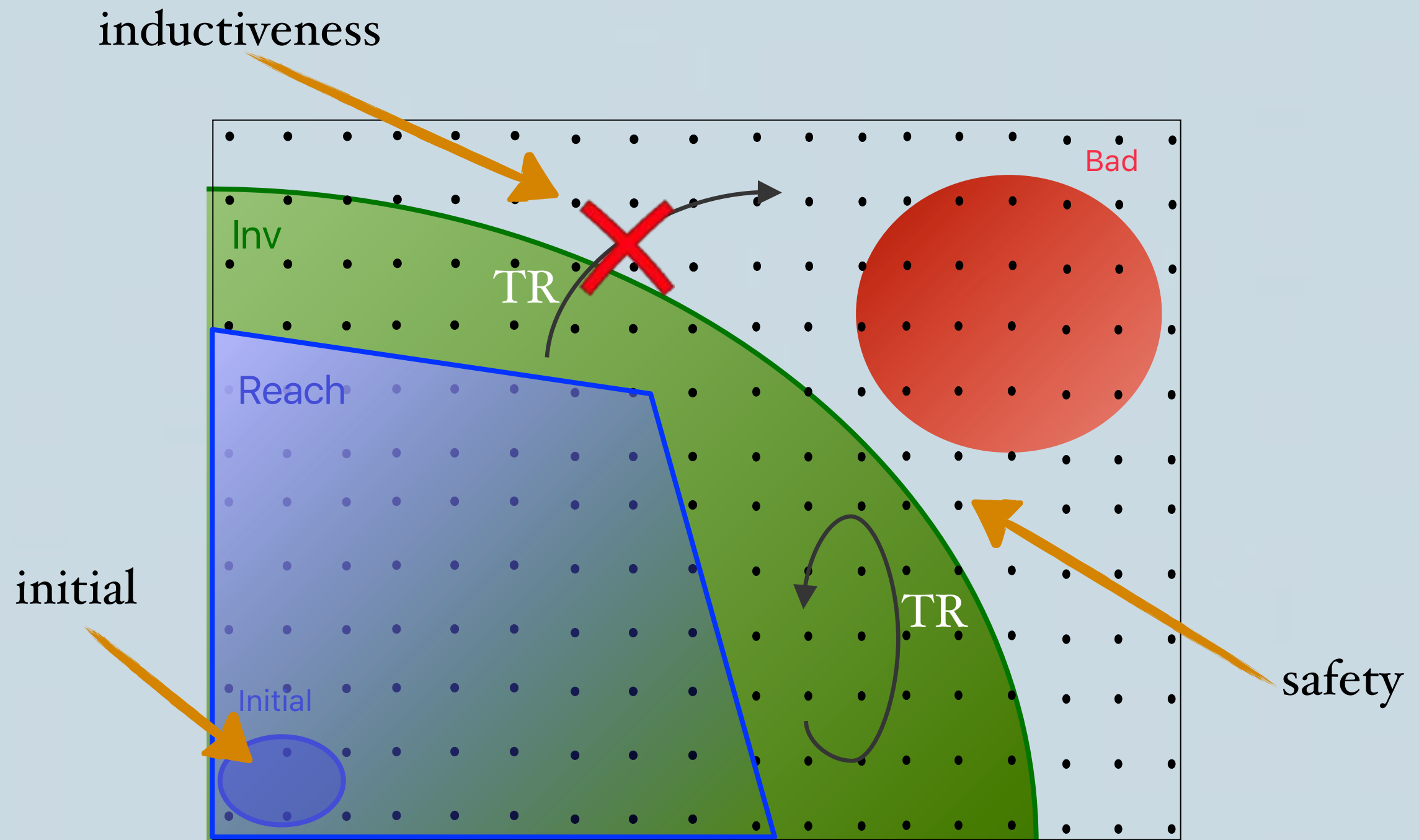
Inductive Invariants



Inductive Invariants

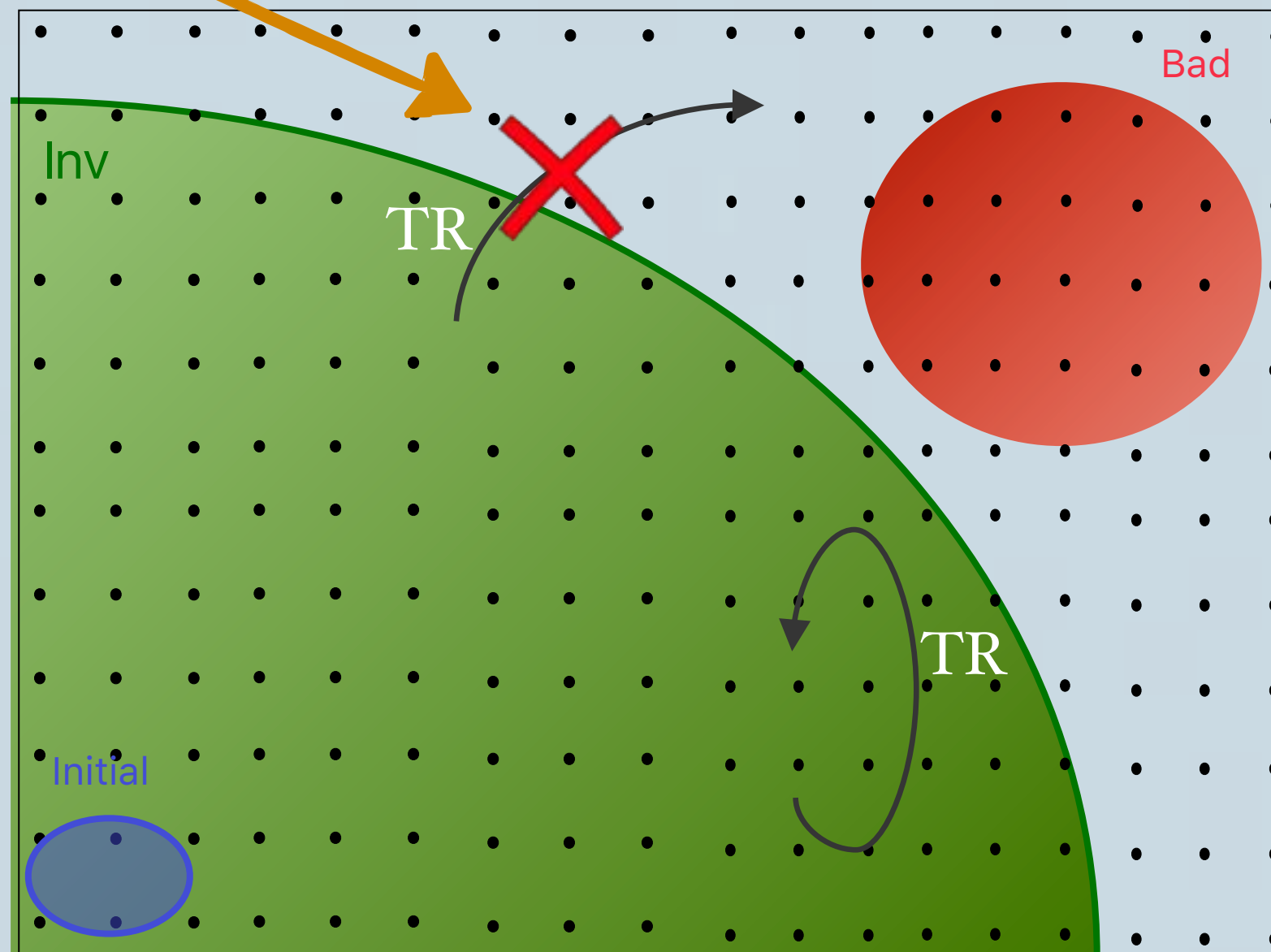


Inductive Invariants



Checking Inductive Invariants

inductiveness



Checking Inductive Invariants

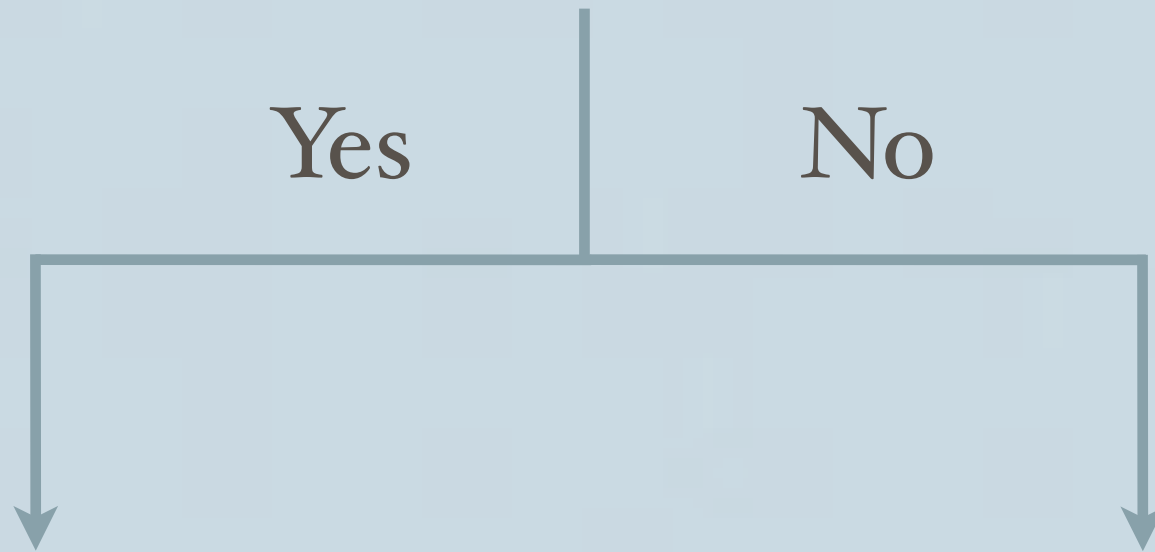
Is **I** inductive for **TR**?

Yes

No

Proof

Violating transition



Checking Inductive Invariants

Is **I** inductive for **TR**?

Yes

No

Proof

Violating transition

Goal: check automatically

Checking Inductive Invariants

$$\mathbf{I}(V) \wedge \mathbf{TR}(V, V') \wedge \neg \mathbf{I}(V')$$

invariant inductive iff formula unsat

Checking Inductive Invariants

$$\mathbf{I}(V) \wedge \mathbf{TR}(V, V') \wedge \neg \mathbf{I}(V')$$

invariant inductive iff formula unsat



Pure First-Order Logic Modeling

+ Quantification

- Arithmetic

- ❖ Shape Analysis [Itzhaky et al. CAV'13, POPL'14, CAV'14, CAV'15]
- ❖ Software-Defined Networks [Ball et al. PLDI'14]
- ❖ Distributed protocols [Padon et al. PLDI'16]
- ❖ Concurrent Modification Errors in Java programs [Frumkin et al. VMCAI'17]

Effectively Propositional Logic (EPR)

- ❖ No function symbols: constants + relations
- ❖ No arithmetic
- ❖ Restricted quantifier prefix: $\exists^* \forall^*$
- ❖ Satisfiability is decidable

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$$\text{TR} = \exists^* \forall^*$$

$$\text{Inv} = \forall^*$$

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$$\text{TR} = \exists^* \forall^*$$

$$\text{Inv} = \forall^*$$

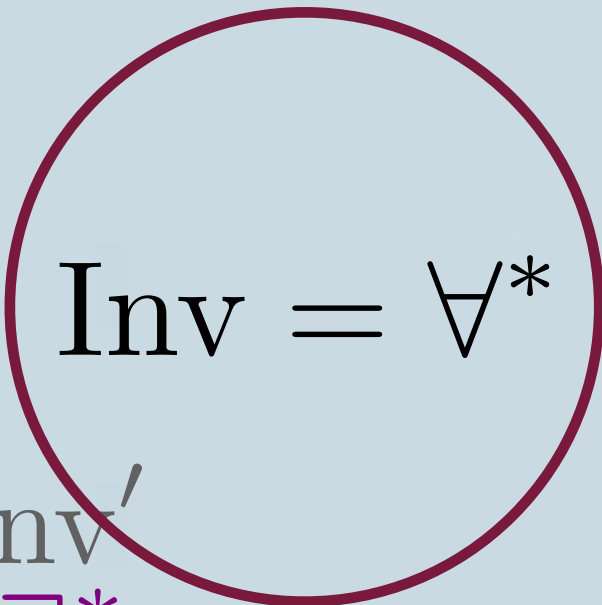
$$\text{Inv} \wedge \text{TR} \wedge \neg \text{Inv}'$$

$\forall^* \wedge \exists^* \forall^* \wedge \exists^*$

Effectively Propositional Logic (EPR)

- ❖ No function symbols: constants + relations
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$$\text{TR} = \exists^* \forall^*$$


$$\text{Inv} = \forall^*$$

$$\text{Inv} \wedge \text{TR} \wedge \neg \text{Inv}'$$

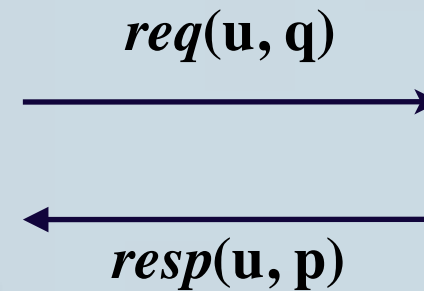
$\forall^* \wedge \exists^* \forall^* \wedge \exists^*$

Example

```
req := ∅; resp := ∅; match := ∅;  
action new_request(u) {  
  q := new request;  
  req := req ∪ {(u, q)}  
}
```

```
action respond(u,q) {  
  assume req(u, q);  
  p := new response;  
  match := match ∪ {(q, p)};  
  resp := resp ∪ {(u, p)}  
}
```

```
action check(u, p) {  
  if resp(u, p) ∧ ¬(∃q. req(u, q) ∧ match(q, p))  
    then abort  
}
```



match(p, q)



$$I = \forall u, p. \text{resp}(u, p) \rightarrow \exists q. \text{req}(u, q) \wedge \text{match}(q, p)$$

No Longer Decidable

$$\text{Inv} = \forall^* \quad \longrightarrow \quad \text{Inv} = \forall^* \exists^*$$

- ❖ Checking inductiveness of $\forall^* \exists^*$ invariants for EPR programs is **undecidable**.

Solving with Quantifier Alternation

$$I \wedge TR \wedge \neg I'$$

unsat

sat

how to find
a proof?

matching loops

how to find
a cex?

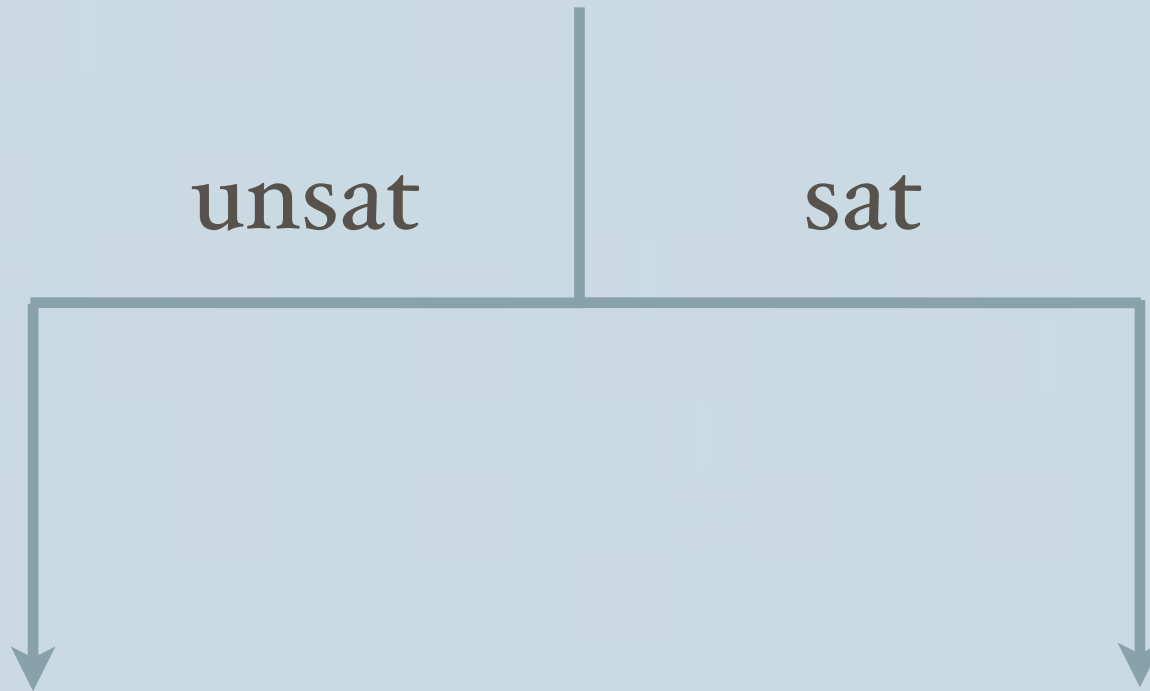
infinite cex

Our Approach: Bounded-Horizon

$$I \wedge TR \wedge \neg I'$$

unsat

sat



Our Approach: Bounded-Horizon

$$I \wedge TR \wedge \neg I'$$

unsat

sat

sound but
incomplete

```
graph TD; A["I ∧ TR ∧ ¬I'"] --> B["unsat"]; A --> C["sat"]; B --> D["sound but incomplete"]; C --> D;
```

Our Approach: Bounded-Horizon

$$I \wedge TR \wedge \neg I'$$

unsat

sat

sound but
incomplete

**powerful as
manual reductions**
to a decidable class
via **instrumentation**

Our Approach: Bounded-Horizon

$$I \wedge TR \wedge \neg I'$$

unsat

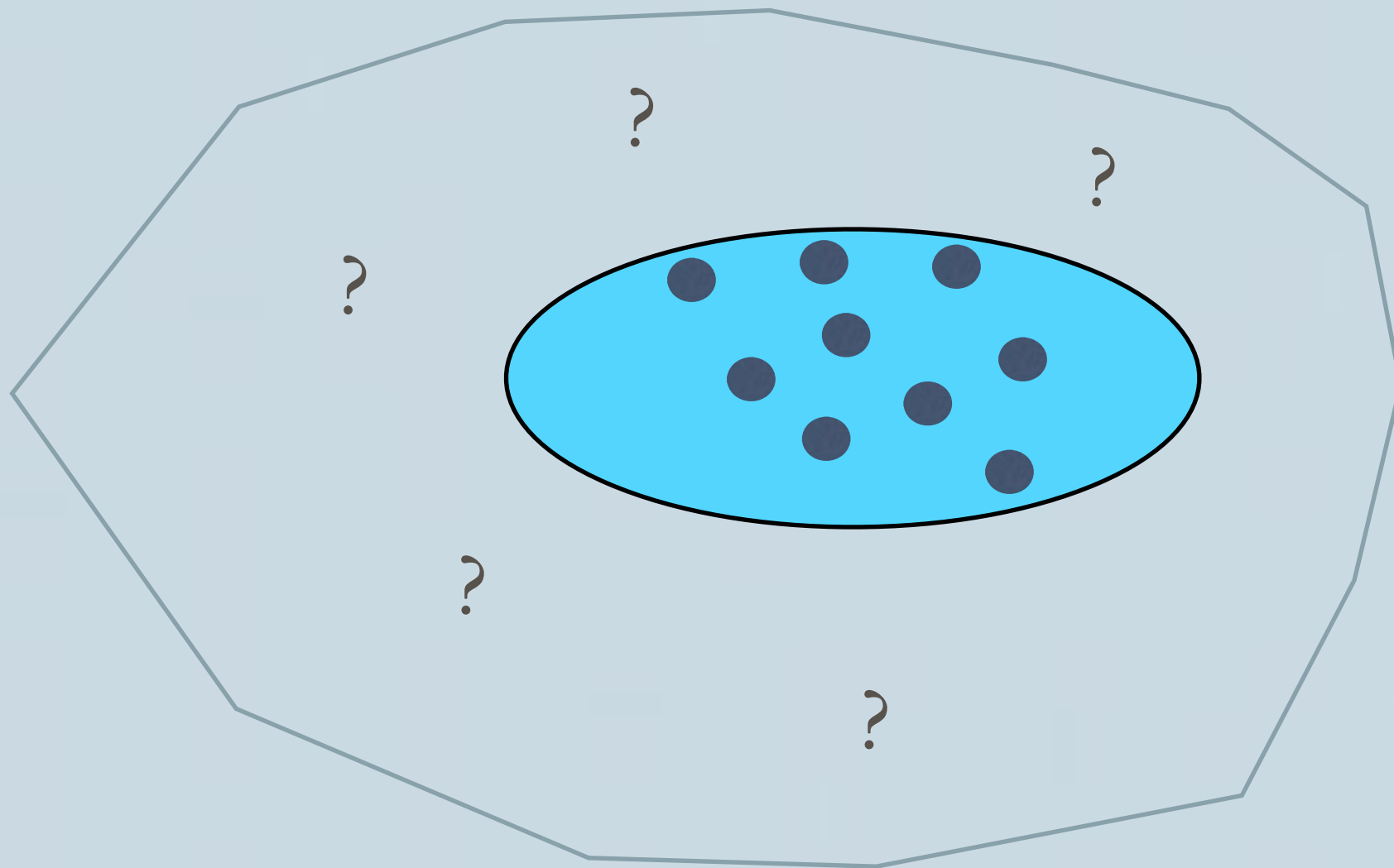
sat

sound but
incomplete

**powerful as
manual reductions**
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via **instrumentation**

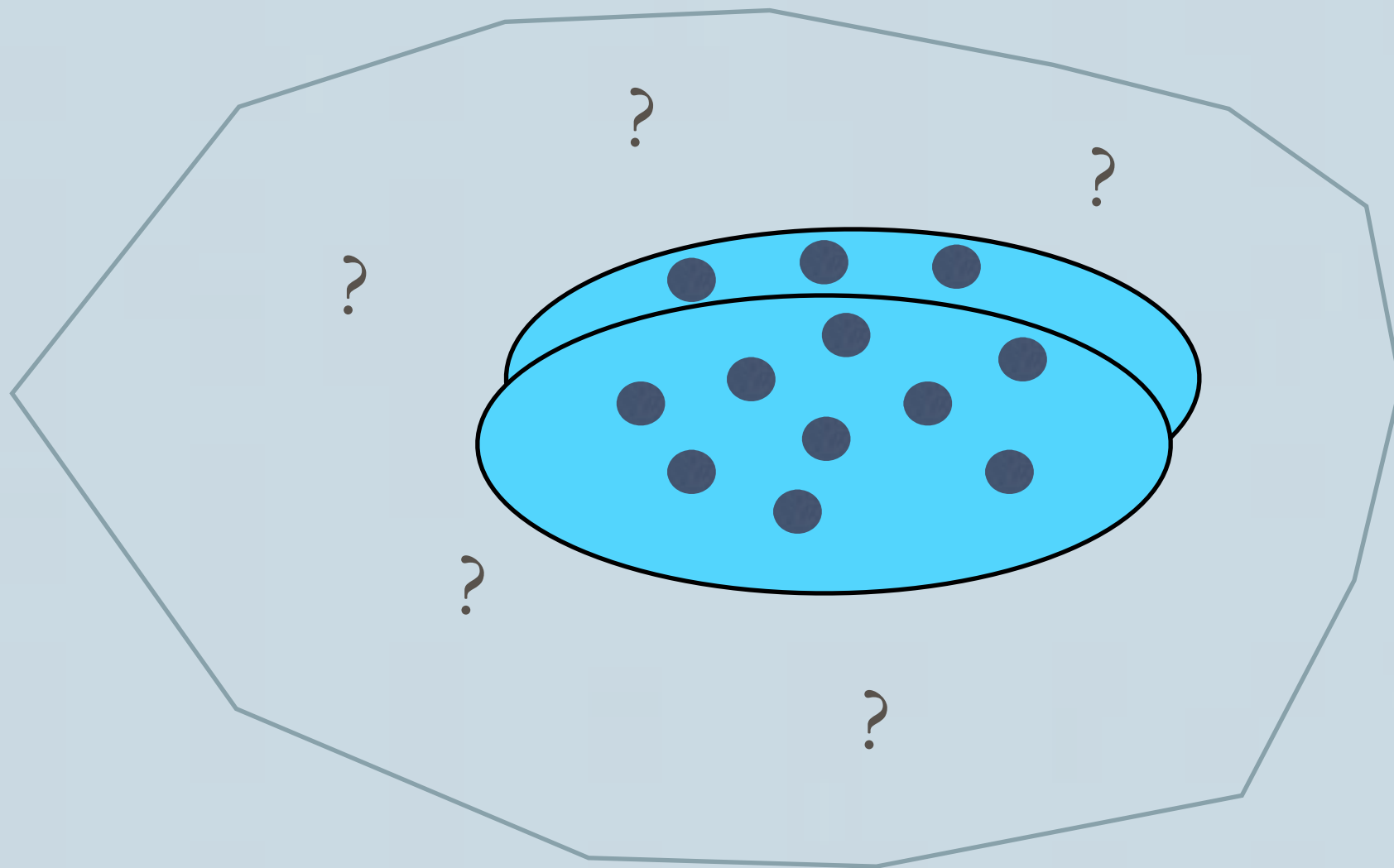
**ability of producing
partial cexs**
to overcome infinite cexs

Bounded-Horizon: Intuition



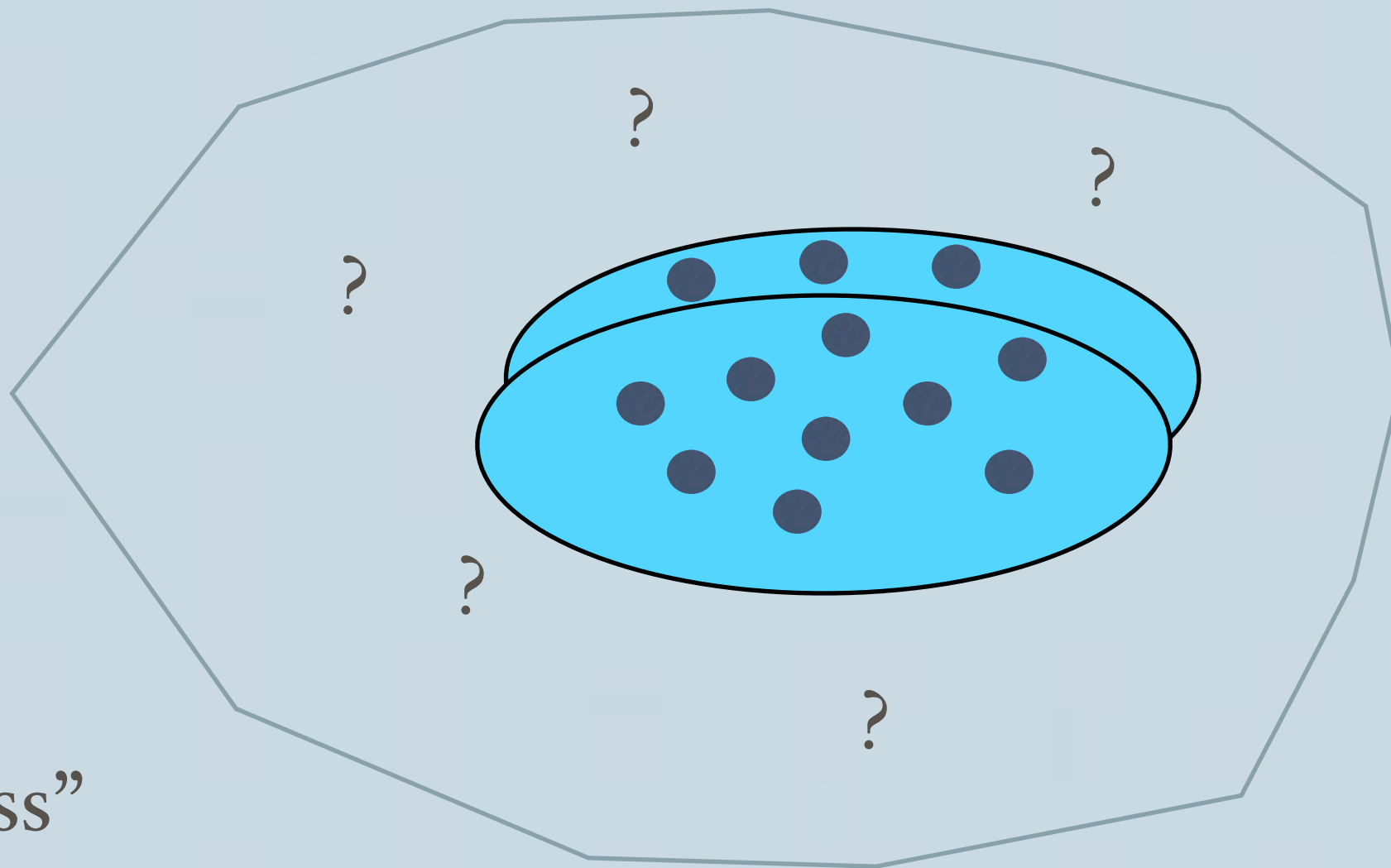
$I \wedge \text{TR} \wedge \neg I'$ is unsat

Bounded-Horizon: Intuition



$I \wedge \text{TR} \wedge \neg I'$ is unsat

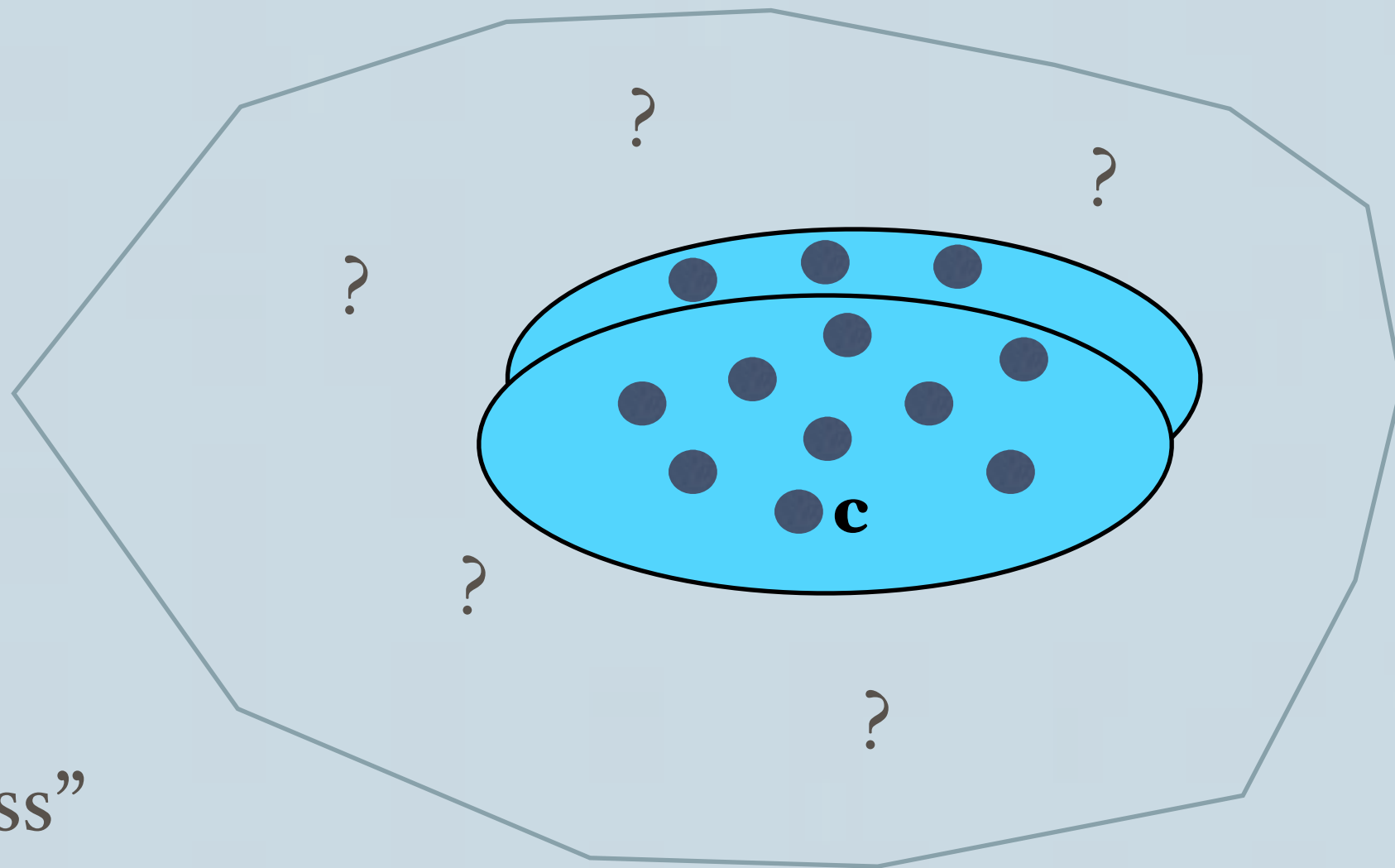
Bounded-Horizon: Intuition



“witness”

$(\forall x. \exists y. \alpha(x, y)) \wedge \text{TR} \wedge \neg I'$ is unsat

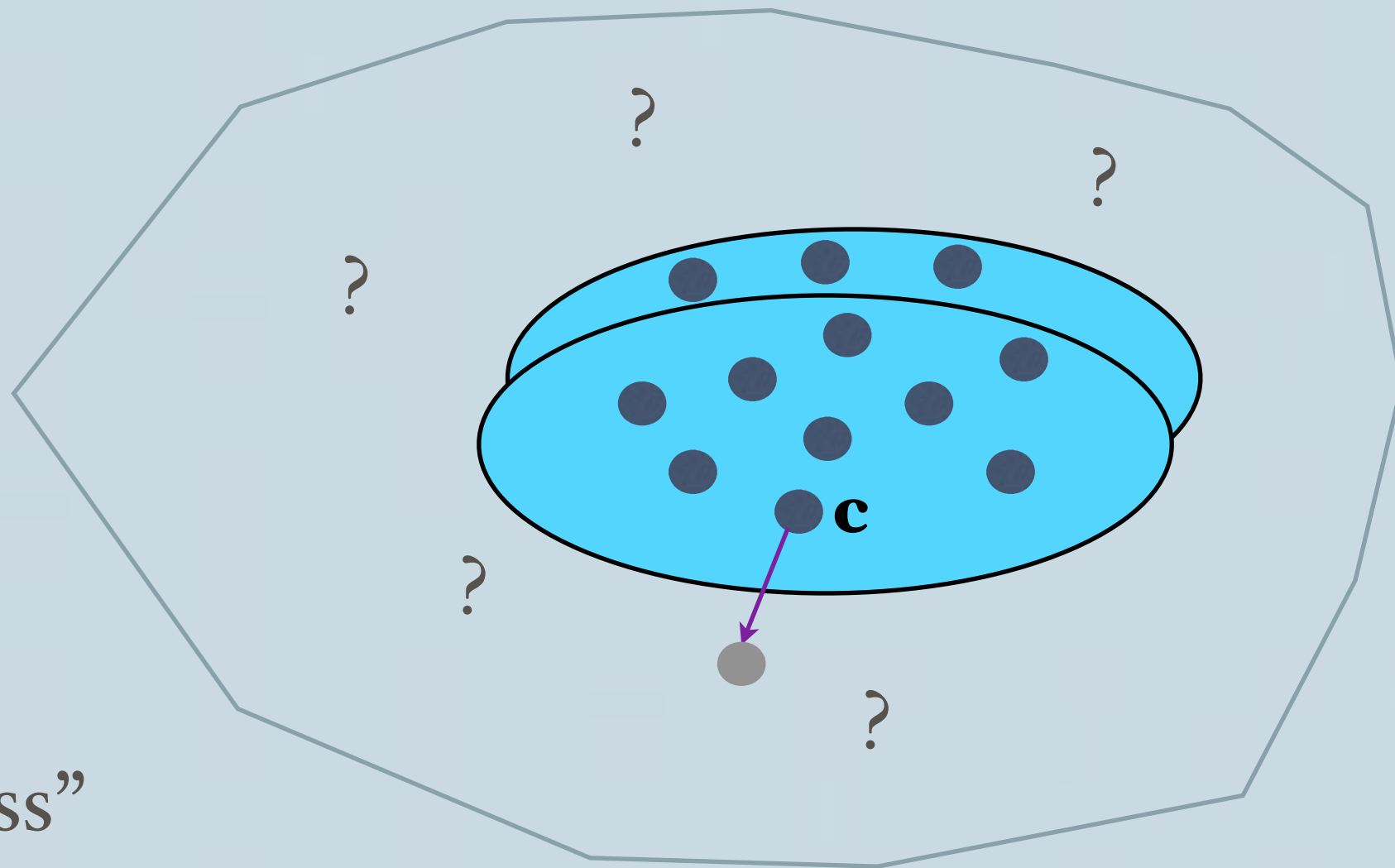
Bounded-Horizon: Intuition



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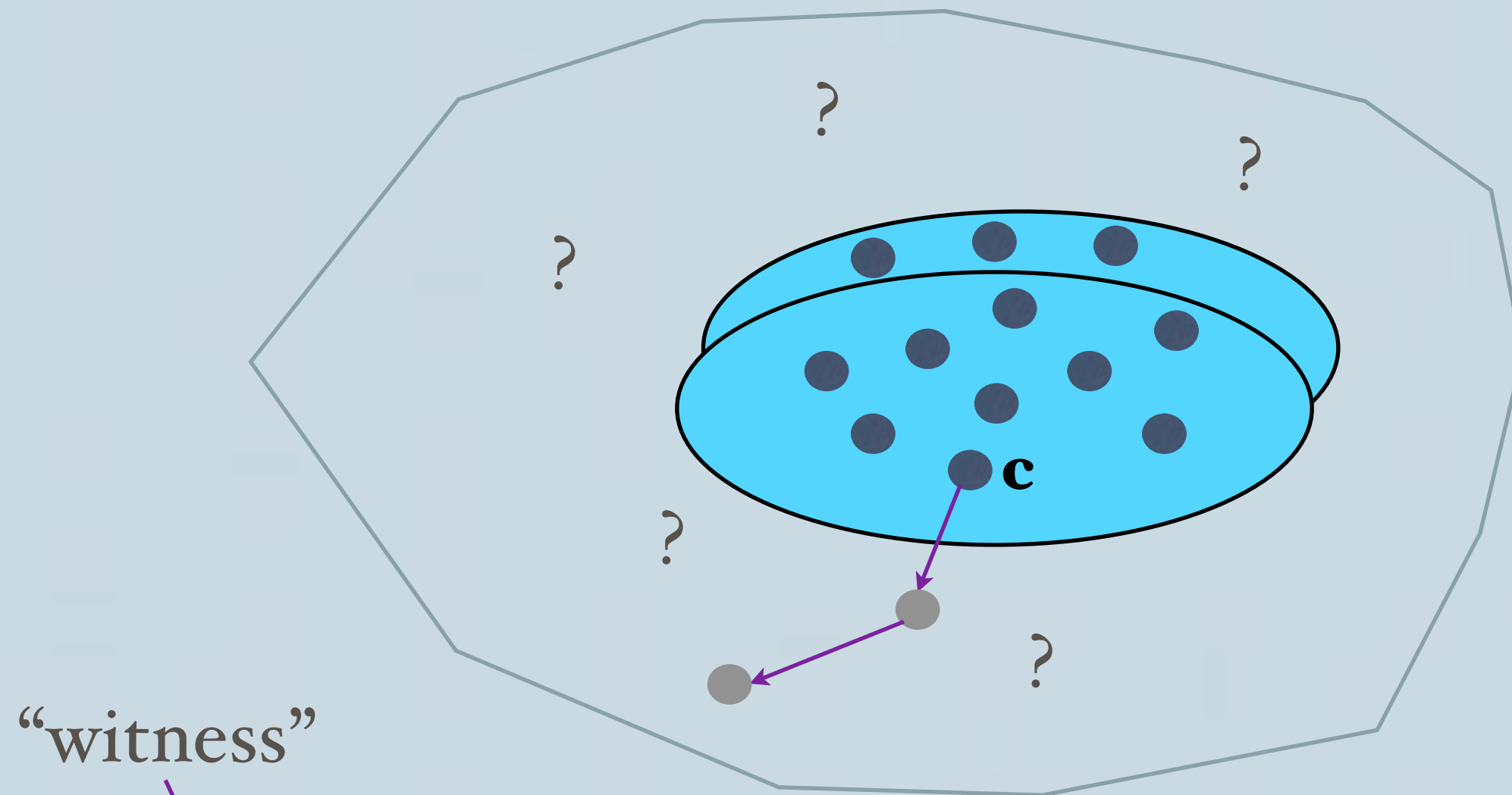
Bounded-Horizon: Intuition



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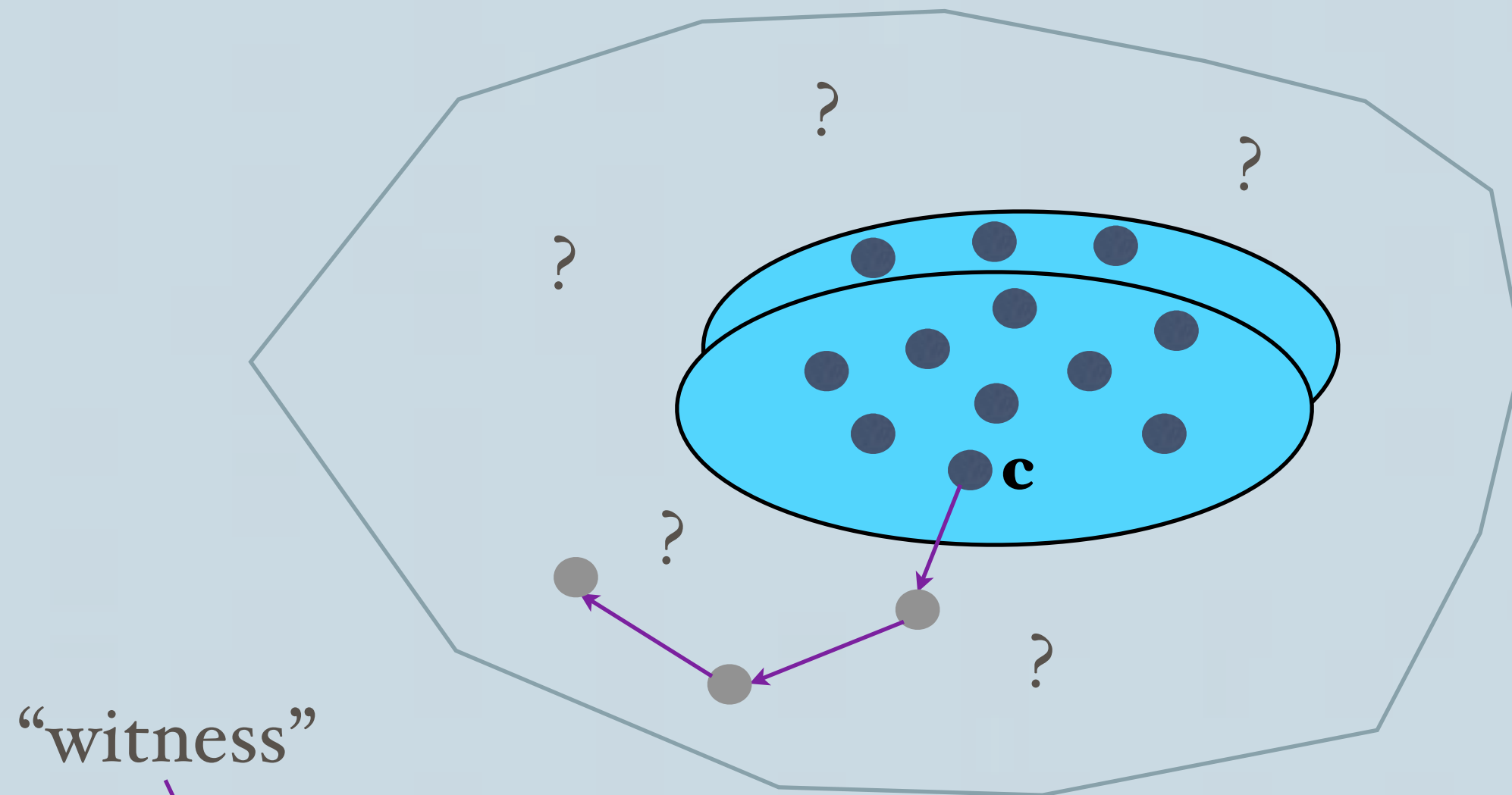
Bounded-Horizon: Intuition



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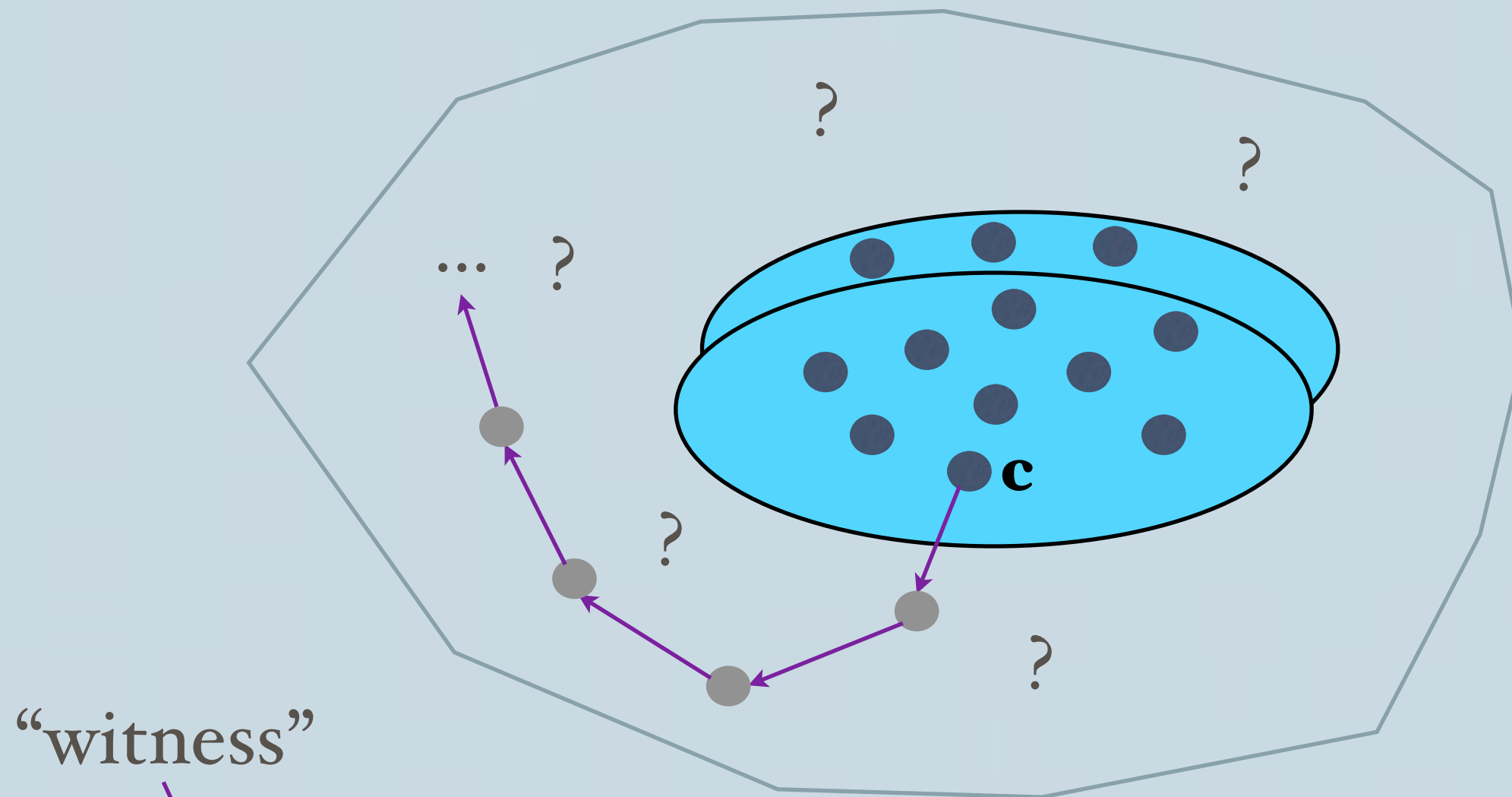
Bounded-Horizon: Intuition



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$(\forall x. \exists y. \alpha(x, y)) \wedge \text{TR} \wedge \neg I'$ is unsat

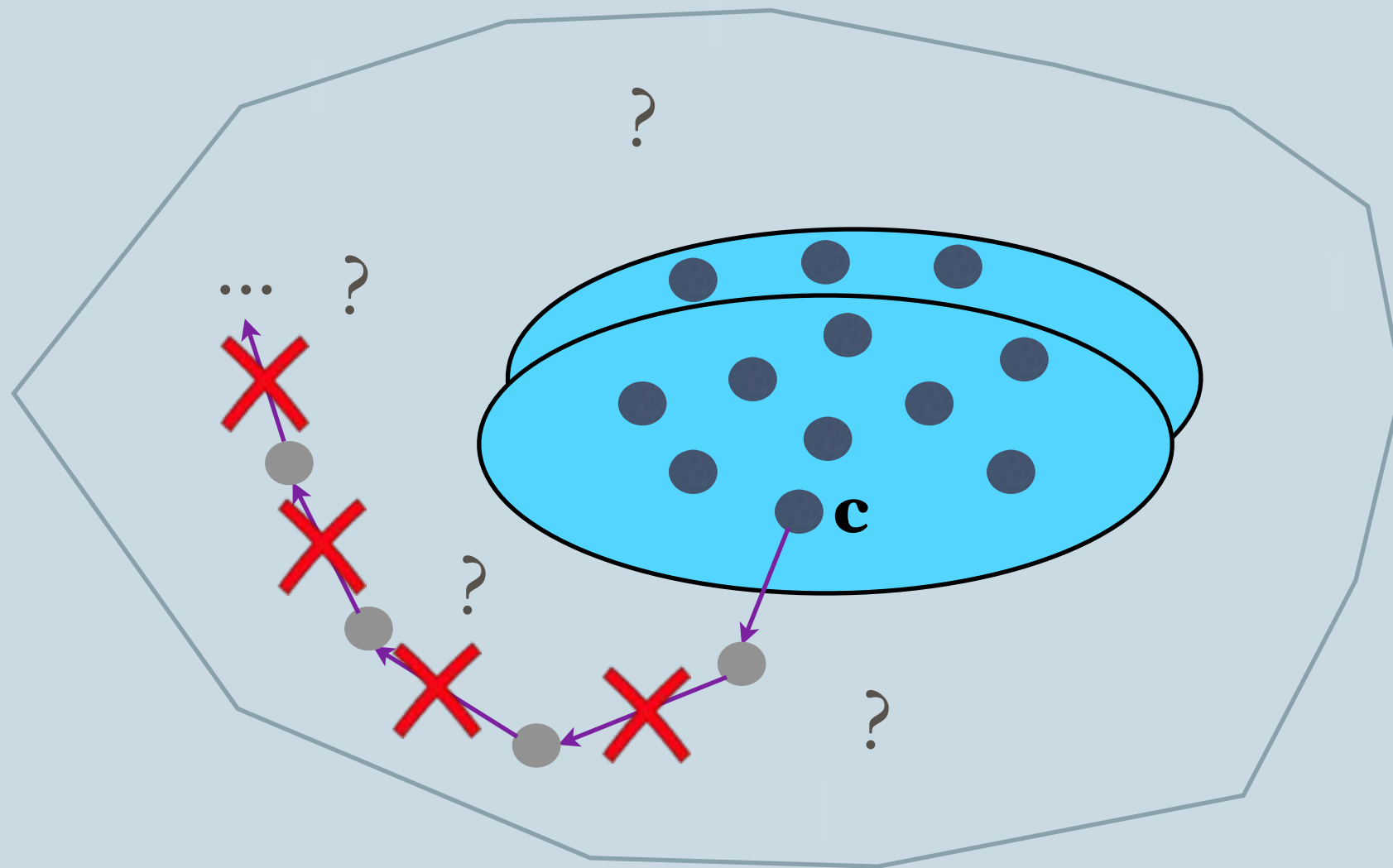
Bounded-Horizon: Intuition



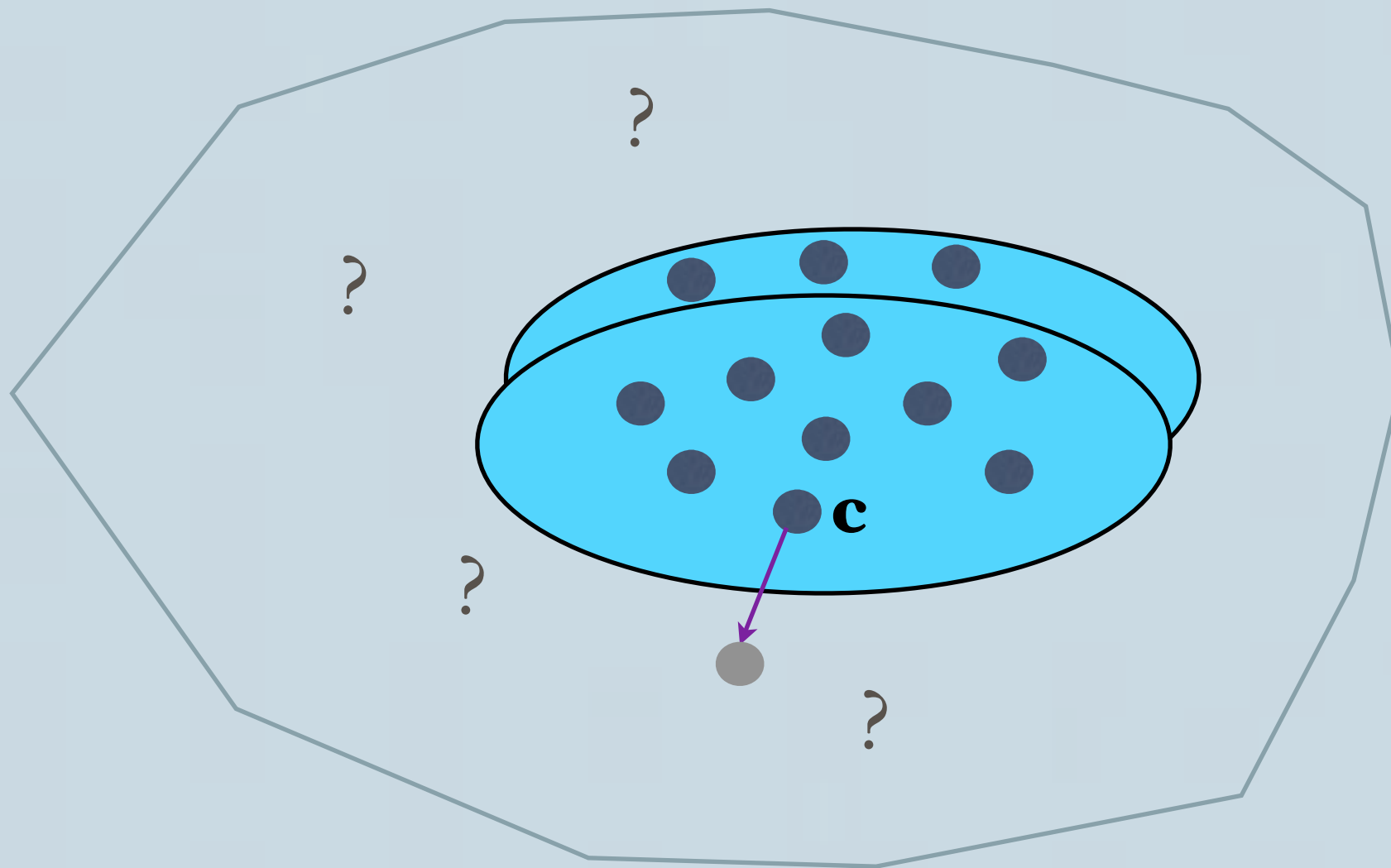
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$(\forall x. \exists y. \alpha(x, y)) \wedge \text{TR} \wedge \neg I'$ is unsat

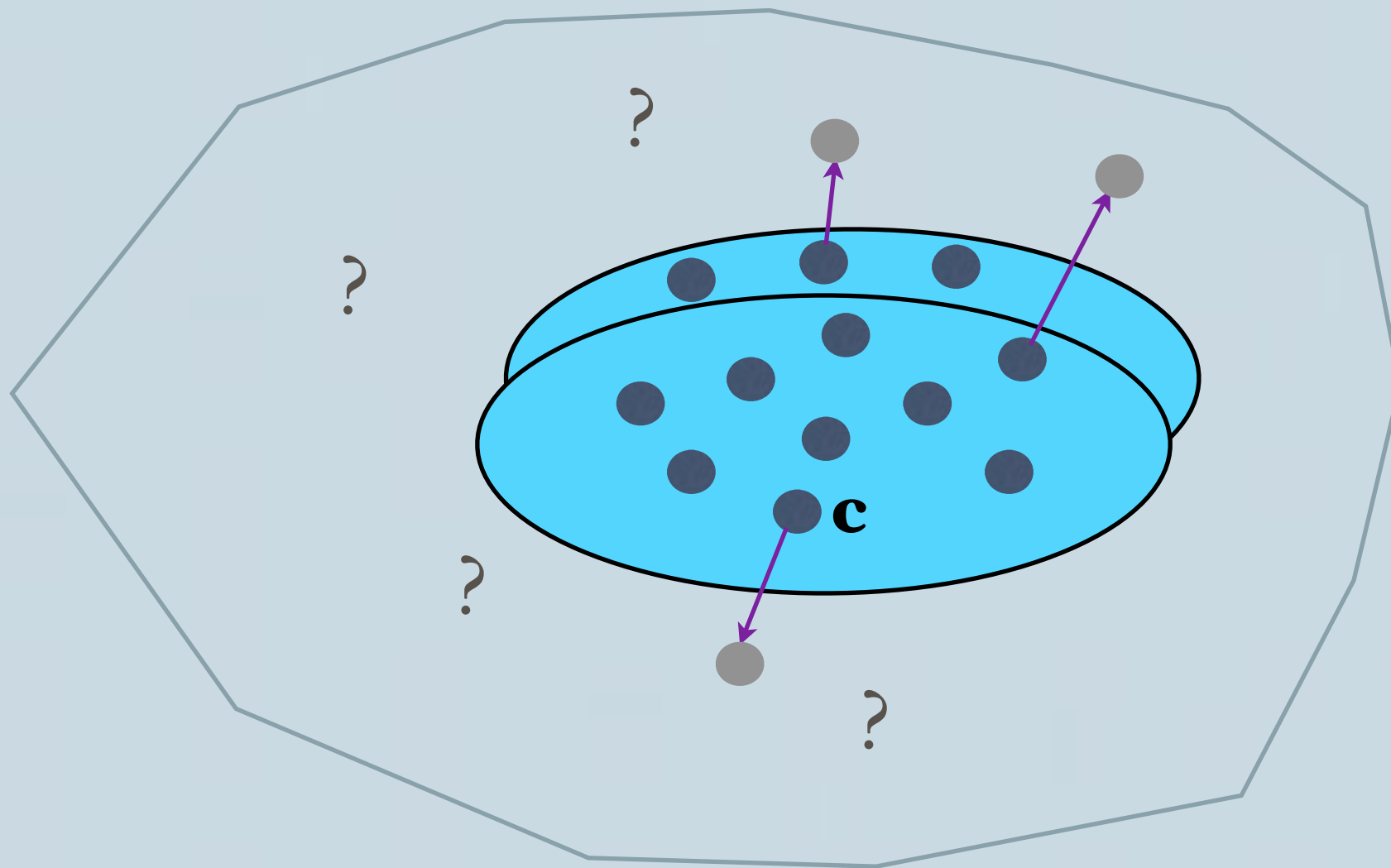
Bounded-Horizon: Intuition



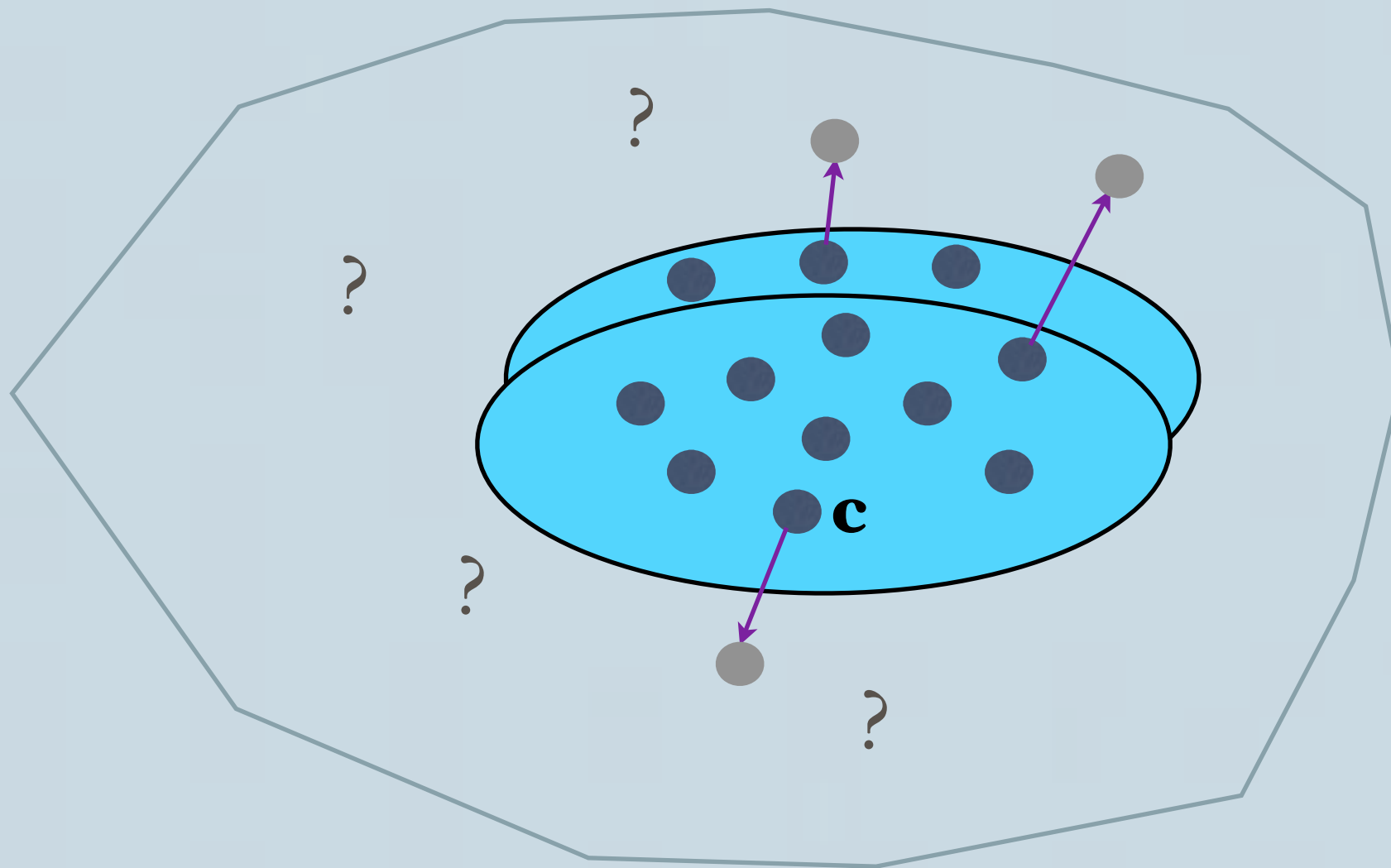
Bounded-Horizon: Intuition



Bounded-Horizon: Intuition

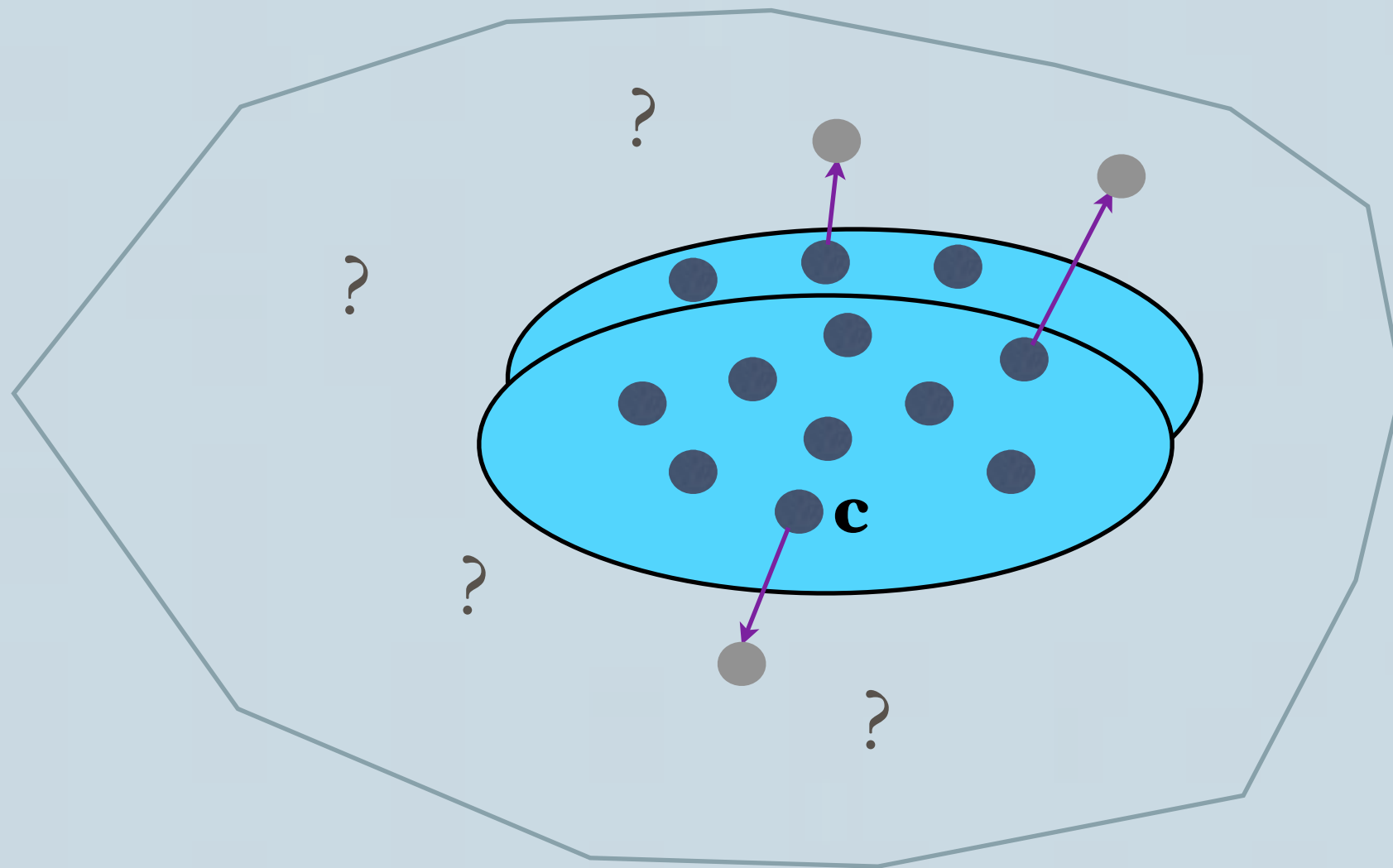


Bounded-Horizon: Intuition



$$\left(\bigwedge_c \exists y. \alpha(c, y) \right) \wedge \text{TR} \wedge \neg I' \text{ is unsat}$$

Bounded-Horizon: Intuition



$\left(\bigwedge_c \exists y. \alpha(c, y) \right) \wedge \text{TR} \wedge \neg I'$ is unsat **decidable!**

Quantifier Instantiation

Check SAT:

$$\forall x. \exists y. \alpha(x, y) \wedge \text{TR} \wedge \neg I'$$

Quantifier Instantiation

Check SAT:

$$\forall x. \exists y. \alpha(x, y)$$

Quantifier Instantiation

Check SAT:

$$\forall x. \exists y. \alpha(x, y)$$



$$\forall x. \alpha(x, f(x))$$

Skolemization

Quantifier Instantiation

Check SAT:

$$\forall x. \exists y. \alpha(x, y)$$



Skolemization

$$\forall x. \alpha(x, f(x))$$



Herbrand's Theorem

$$\left\{ \alpha(t, f(t)) \mid t \text{ ground term} \right\}$$

Bounded Instantiation

Check SAT:

$$\forall x. \exists y. \alpha(x, y)$$

Skolemization

$$\forall x. \alpha(x, f(x))$$

Herbrand's Theorem

decidable! $\left\{ \alpha(t, f(t)) \mid t \text{ ground term} \right\}$

$\leq k$

Bounded Instantiation

Check SAT:

$$\forall x. \exists y. \alpha(x, y)$$

Skolemization

$$\forall x. \alpha(x, f(x))$$

Herbrand's Theorem

decidable! $\left\{ \alpha(t, f(t)) \mid t \text{ ground term} \right\}$

$\leq k$

sound but incomplete

Our Approach: Bounded-Horizon

$$I \wedge TR \wedge \neg I'$$

unsat

sat

how to find
a proof?

**at least as powerful
as instrumentation**

how to find
a cex?

ability of producing
partial cexs

Reduction by Instrumentation

$$\begin{array}{ccc} \text{TR} & & \widehat{\text{TR}} \\ \text{Inv} = \forall^* \exists^* & \longrightarrow & \widehat{\text{Inv}} = \forall^* \end{array}$$

- S. Sagiv, T. W. Reps, and R. Wilhelm. Parametric Shape Analysis via 3-Valued Logic (TOPLAS'02)
- S. Itzhaky, A. Banerjee, N. Immerman, A. Nanevski, and M. Sagiv. Effectively-Propositional Reasoning About Reachability in Linked Data Structures (CAV'13)
- A. Karbyshev, N. Bjørner, S. Itzhaky, N. Rinetzky, and S. Shoham. Property-Directed Inference of Universal Invariants or Proving their Absence (CAV'15)
- O. Padon, K. L. McMillan, A. Panda, M. Sagiv, and S. Shoham. Ivy: Safety Verification by Interactive Generalization (PLDI'16)

Reduction by Instrumentation

$$\begin{array}{ccc} \text{TR} & \text{auxiliary state for} & \widehat{\text{TR}} \\ & \text{derived relation} & \\ \text{Inv} = \forall^* \exists^* & \xrightarrow{\text{“}r(x) \equiv \psi(x)\text{”}} & \widehat{\text{Inv}} = \forall^* \end{array}$$

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Instrumentation Example

$\text{req} := \emptyset; \text{resp} := \emptyset; \text{match} := \emptyset;$

action new request(u) {

$q := \text{new request};$

$\text{req} := \text{req} \cup \{(u, q)\}$

}

$I = \forall u, p. \text{resp}(u, p) \rightarrow \exists q. \text{req}(u, q) \wedge \text{match}(q, p)$

action respond(u,q) {

 assume $\text{req}(u, q);$

$p := \text{new response};$

$\text{match} := \text{match} \cup \{(q, p)\};$

$\text{resp} := \text{resp} \cup \{(u, p)\}$

}

action check(u, p) {

 if $\text{resp}(u, p) \wedge \neg(\exists q. \text{req}(u, q) \wedge \text{match}(q, p))$

 then abort

}

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$I = \forall u, p. \text{resp}(u, p) \rightarrow \boxed{\exists q. \text{req}(u, q) \wedge \text{match}(q, p)}$

action respond(u,q) {

 assume $\text{req}(u, q);$

$p := \text{new response};$

$\text{match} := \text{match} \cup \{(q, p)\};$

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 then abort

}

Instrumentation Example

```
req := ∅; resp := ∅; match := ∅;  
action new request(u) {  
  q := new request;  
  req := req ∪ {(u, q)}
```

$r(u, p) \text{ “}\equiv\text{” } \exists q. req(u, q) \wedge match(q, p)$

```
}
```

$I = \forall u, p. resp(u, p) \rightarrow \boxed{\exists q. req(u, q) \wedge match(q, p)}$

```
action respond(u, q) {  
  assume req(u, q);  
  p := new response;  
  match := match ∪ {(q, p)};
```

```
  resp := resp ∪ {(u, p)}  
}
```

```
action check(u, p) {  
  if resp(u, p)  $\wedge$   $\neg(\exists q. req(u, q) \wedge match(q, p))$   
    then abort  
}
```

Instrumentation Example

req := $\emptyset$; resp := $\emptyset$; match := $\emptyset$;

action new request(u) {

 q := new request;

 req := req $\cup$ {(u, q)}

}

action respond(u,q) {

 assume req(u, q);

 p := new response;

 match := match $\cup$ {(q, p)};

 resp := resp $\cup$ {(u, p)}

}

action check(u, p) {

 if resp(u, p) $\wedge$ $\neg(\exists q. req(u, q) \wedge match(q, p))$

 then abort

}

$r(u, p) \text{ “}\equiv\text{” } \exists q. req(u, q) \wedge match(q, p)$

$I = \forall u, p. resp(u, p) \rightarrow \boxed{\exists q. req(u, q) \wedge match(q, p)}$

$I = \forall u, p. resp(u, p) \rightarrow r(u, p)$

Instrumentation Example

```
req := ∅; resp := ∅; match := ∅;  
action new request(u) {  
  q := new request;  
  req := req ∪ {(u, q)}
```

```
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  assume req(u, q);  
  p := new response;  
  match := match ∪ {(q, p)};
```

```
  resp := resp ∪ {(u, p)}  
}
```

```
action check(u, p) {  
  if resp(u, p) ∧ ¬(∃q. req(u, q) ∧ match(q, p))  
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```

$r(u, p) \text{ “}\equiv\text{” } \exists q. req(u, q) \wedge match(q, p)$

$I = \forall u, p. resp(u, p) \rightarrow \boxed{\exists q. req(u, q) \wedge match(q, p)}$

$I = \forall u, p. resp(u, p) \rightarrow \boxed{r(u, p)}$

Instrumentation Example

req := $\emptyset$; resp := $\emptyset$; match := $\emptyset$;

action new request(u) {

 q := new request;

 req := req $\cup$ {(u, q)}

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$I = \forall u, p. resp(u, p) \rightarrow r(u, p)$

Instrumentation Example

req := $\emptyset$; resp := $\emptyset$; match := $\emptyset$;

action new request(u) {

 q := new request;

 req := req $\cup$ {(u, q)}

 r := r $\cup$ {(u,y) | match(q,y)}

}

$r(u, p) \text{ “}\equiv\text{” } \exists q. req(u, q) \wedge match(q, p)$

action respond(u,q) {

 assume req(u, q);

 p := new response;

 match := match $\cup$ {(q, p)};

 r := r $\cup$ {(x,p) | req(x,q)};

 resp := resp $\cup$ {(u, p)}

}

$I = \forall u, p. resp(u, p) \rightarrow \exists q. req(u, q) \wedge match(q, p)$

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 p := new response;

 match := match $\cup$ {(q, p)};

 r := r $\cup$ {(x,p) | req(x,q)};

 resp := resp $\cup$ {(u, p)}

}

$I = \forall u, p. resp(u, p) \rightarrow r(u, p)$

action check(u, p) {

 if resp(u, p) $\wedge \neg(\exists q. req(u, q) \wedge match(q, p))$

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$I = \forall u, p. resp(u, p) \rightarrow \exists q. req(u, q) \wedge match(q, p)$

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action respond(u,q) {

 assume req(u, q);

 p := new response;

 match := match $\cup$ {(q, p)};

 r := r $\cup$ {(x,p) | req(x,q)};

 resp := resp $\cup$ {(u, p)}

}

$I = \forall u, p. resp(u, p) \rightarrow r(u, p)$

action check(u, p) {

~~if resp(u, p) $\wedge$ $\neg(\exists q. req(u, q) \wedge match(q, p))$~~ if resp(u, p) $\wedge$ $\neg r(u, p)$

 then abort

}

Instrumentation Example

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req := ∅; resp := ∅; match := ∅;
action new request(u) {
  q := new request;
  req := req ∪ {(u, q)}
  r := r ∪ {(u,y) | match(q,y)}
}

```

$r(u, p) \text{ “}\equiv\text{” } \exists q. req(u, q) \wedge match(q, p)$

$I = \forall u, p. resp(u, p) \rightarrow \exists q. req(u, q) \wedge match(q, p)$

```

action respond(u,q) {
  assume req(u, q);
  p := new response;
  match := match ∪ {(q, p)};
  r := r ∪ {(x,p) | req(x,q)};
  resp := resp ∪ {(u, p)}
}

```

$I = \forall u, p. resp(u, p) \rightarrow r(u, p)$

```

action check(u, p) {
  if resp(u, p) ∧ ¬(∃q. req(u, q) ∧ match(q, p)) if resp(u, p) ∧ ¬r(u, p)
    then abort
}

```

Possibly unsound!

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$\text{TR}, \widehat{\text{TR}}$ have the same existentials

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**using Bounded-Horizon
with bound 1 or 2**

Bounded-Horizon vs. Instrumentation

Given TR , $I \in \forall^* \exists^*$ **if exists** sound instrumentation
without additional existentials

$$I = \widehat{I}[\psi/r] \quad \widehat{I} \in \forall^* \quad \widehat{I} \text{ is inductive for } \widehat{\text{TR}}$$

Then

❖ Theorem:

For ψ that is a Boolean combination of $\forall^*$ and $\exists^*$,
for proving that I is inductive for TR it suffices to use
Bounded-Horizon with bound=2.

Bounded-Horizon vs. Instrumentation

Given TR , $I \in \forall^* \exists^*$ **if exists** sound instrumentation
without additional existentials

$$I = \widehat{I}[\psi/r] \quad \widehat{I} \in \forall^* \quad \widehat{I} \text{ is inductive for } \widehat{\text{TR}}$$

Then

❖ Theorem:

For $\psi \in \exists^*$ and r appears only positively in the invariant
(or $\psi \in \forall^*$ and r appears only negatively in the invariant),

Bounded-Horizon with bound=1 suffices.

❖ Invariants that are combinations of $\forall^* \exists^*$ and $\exists^* \forall^*$ also handled

Our Approach: Bounded-Horizon

$$I \wedge TR \wedge \neg I'$$

unsat

sat

how to find
a proof?

at least as powerful
as instrumentation

how to find
a cex?

**ability of producing
partial cexs**

Infinite Counterexample

$$I \wedge \text{TR} \wedge \neg I'$$

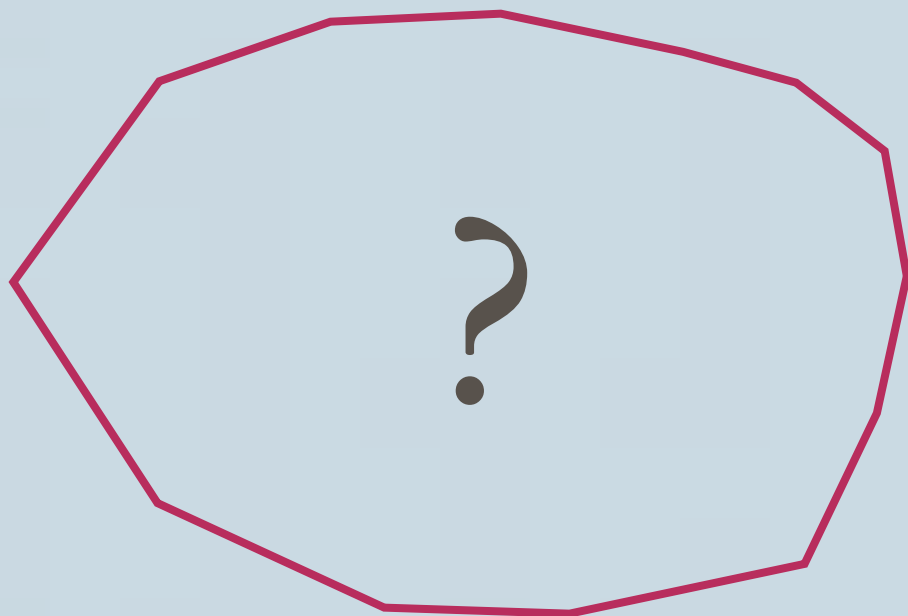
has **infinite** model
but **no finite** models

Infinite Counterexample

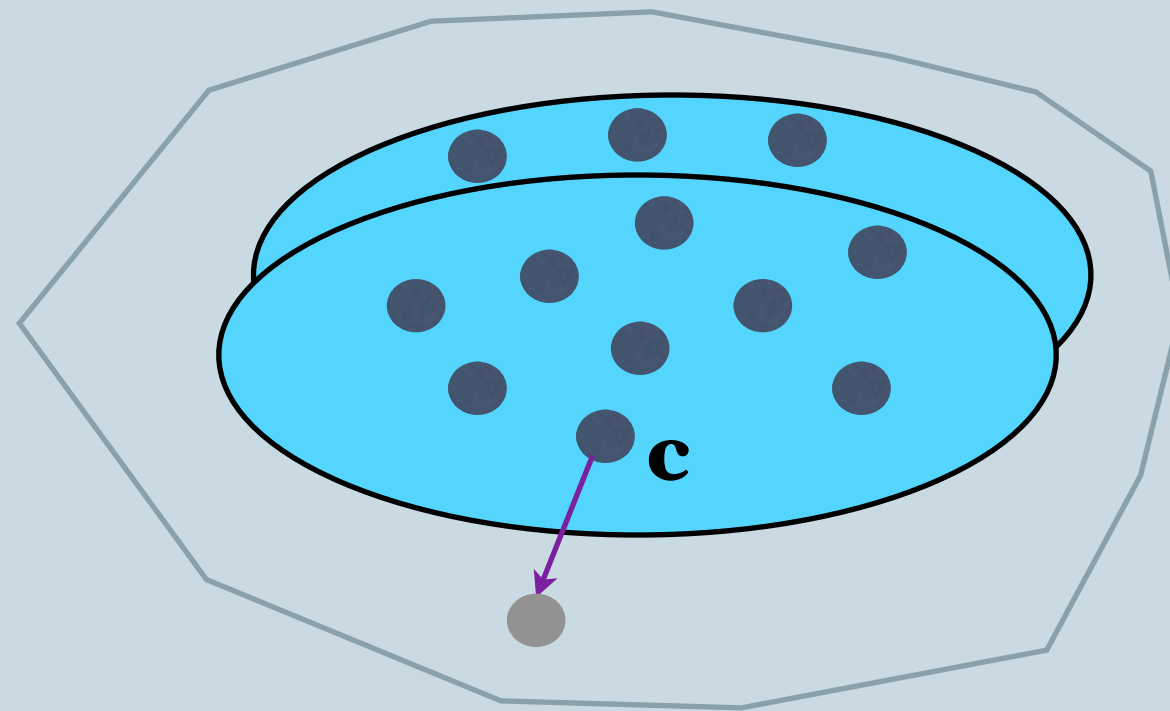
$$I \wedge \text{TR} \wedge \neg I'$$

has **infinite** model
but **no finite** models

usually **no support** for **infinite cexs**
in general first-order logic

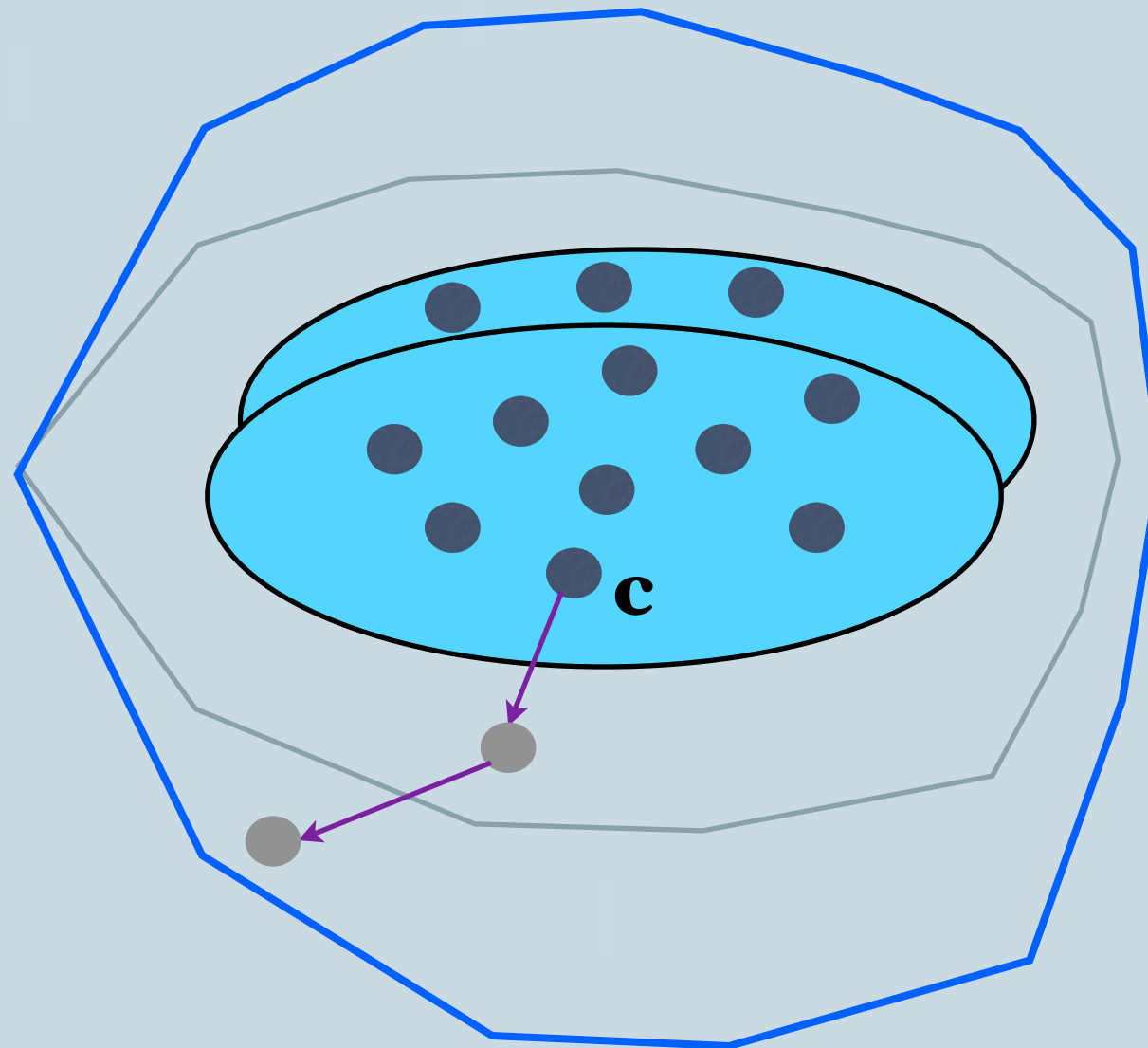


Partial Models



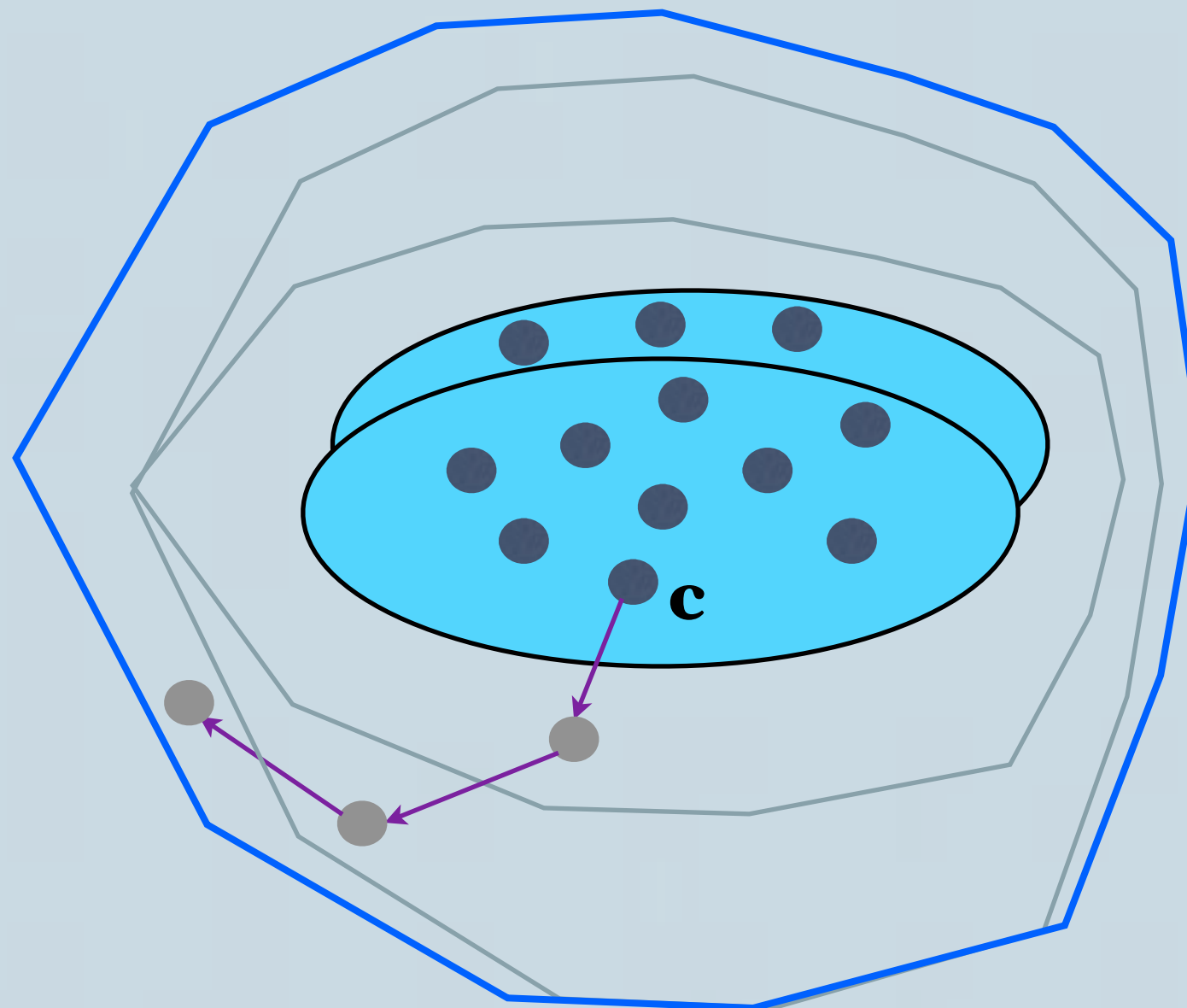
Bound $\mathbf{I}$

Partial Models



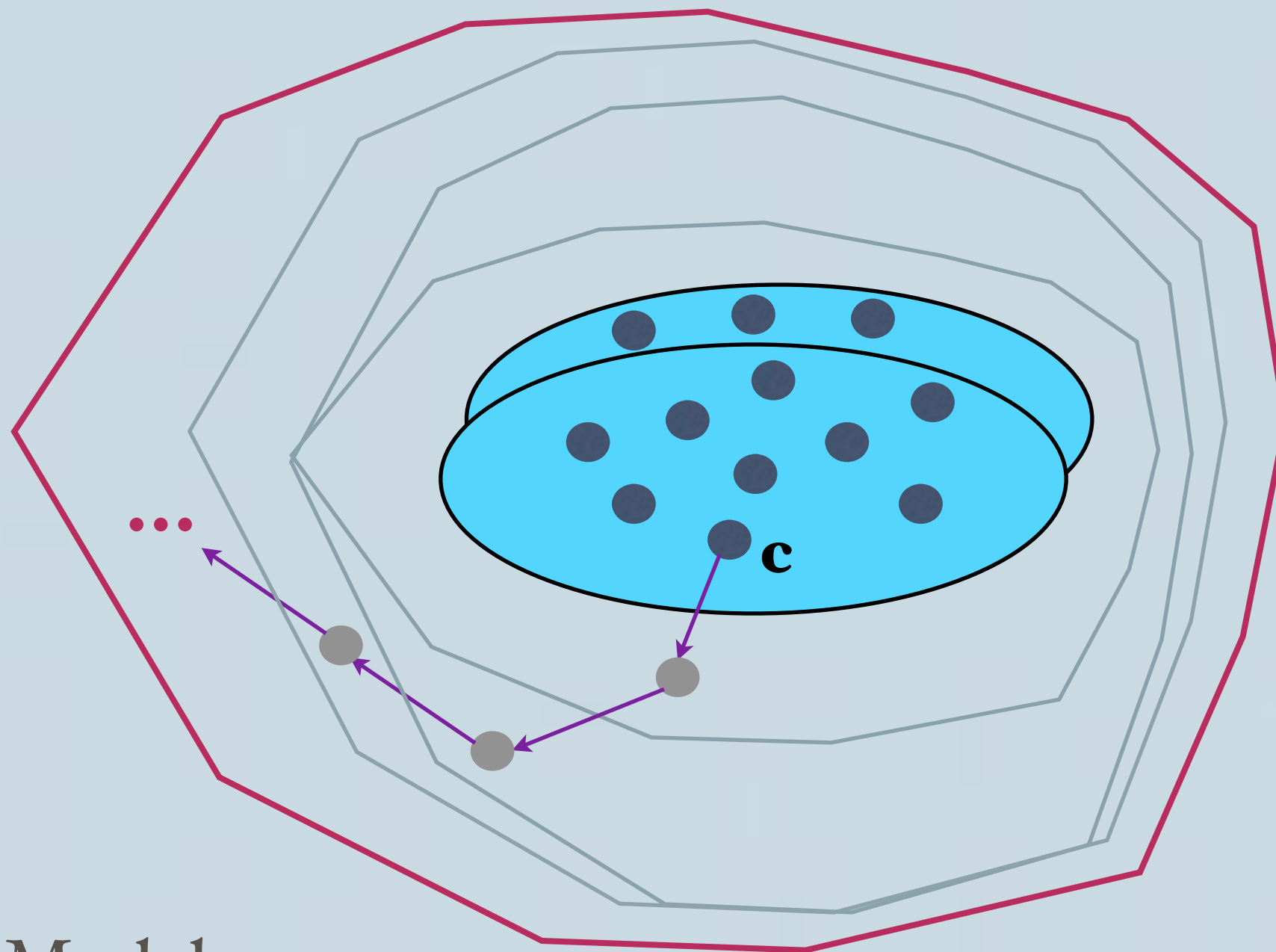
Bound 2

Partial Models



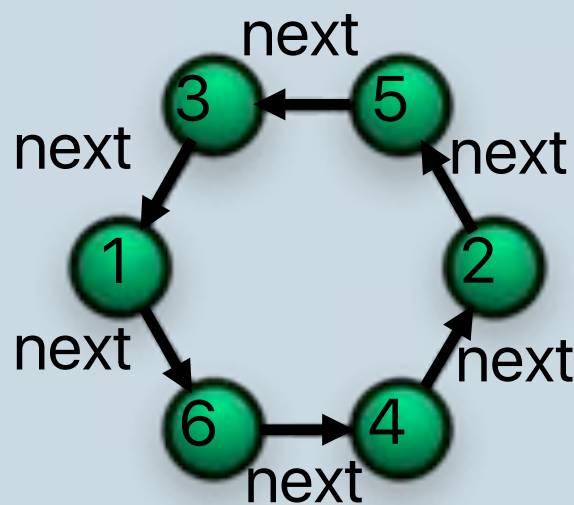
Bound 3

Partial Models



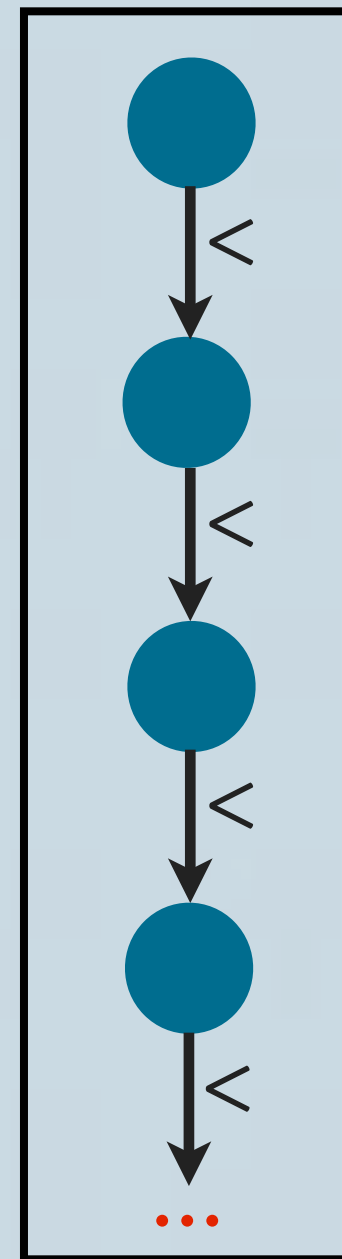
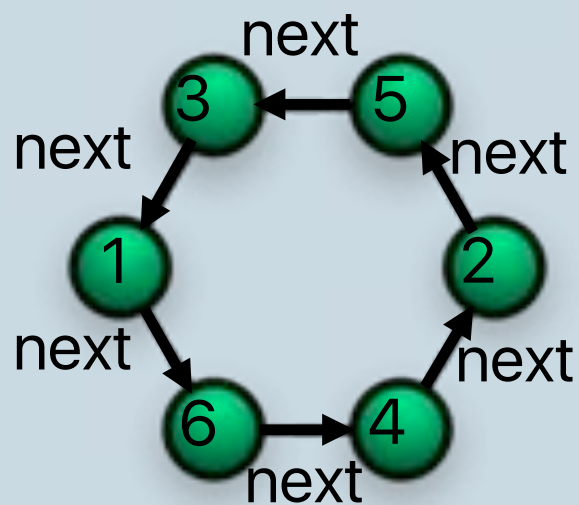
Total Model

Example: Leader Election in a Ring



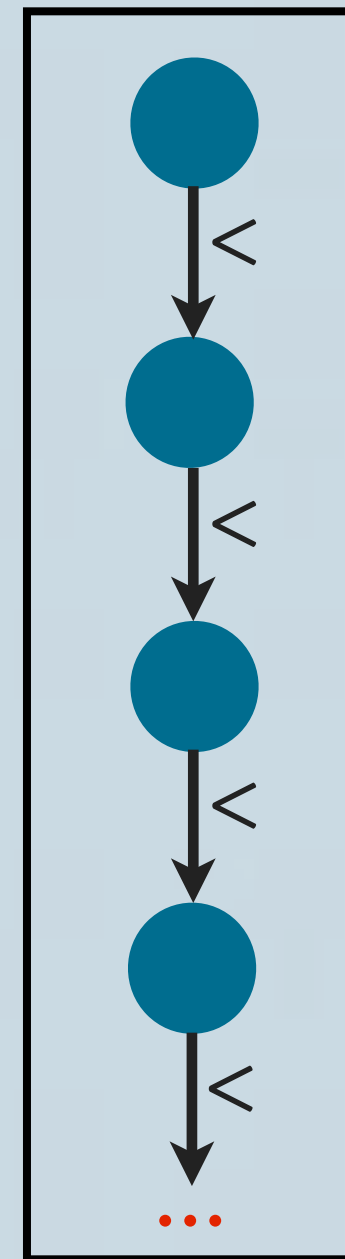
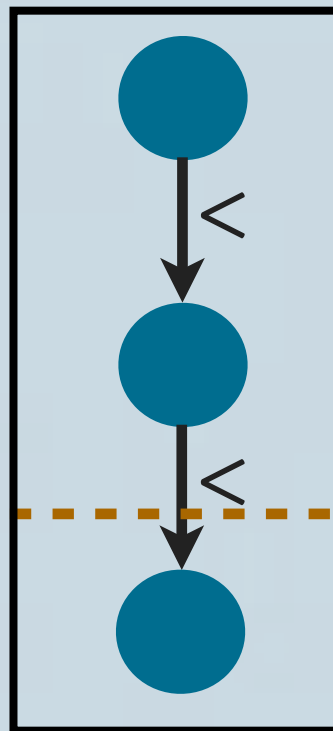
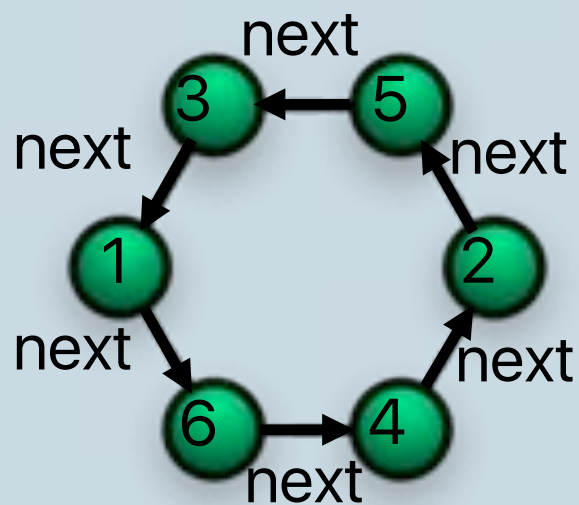
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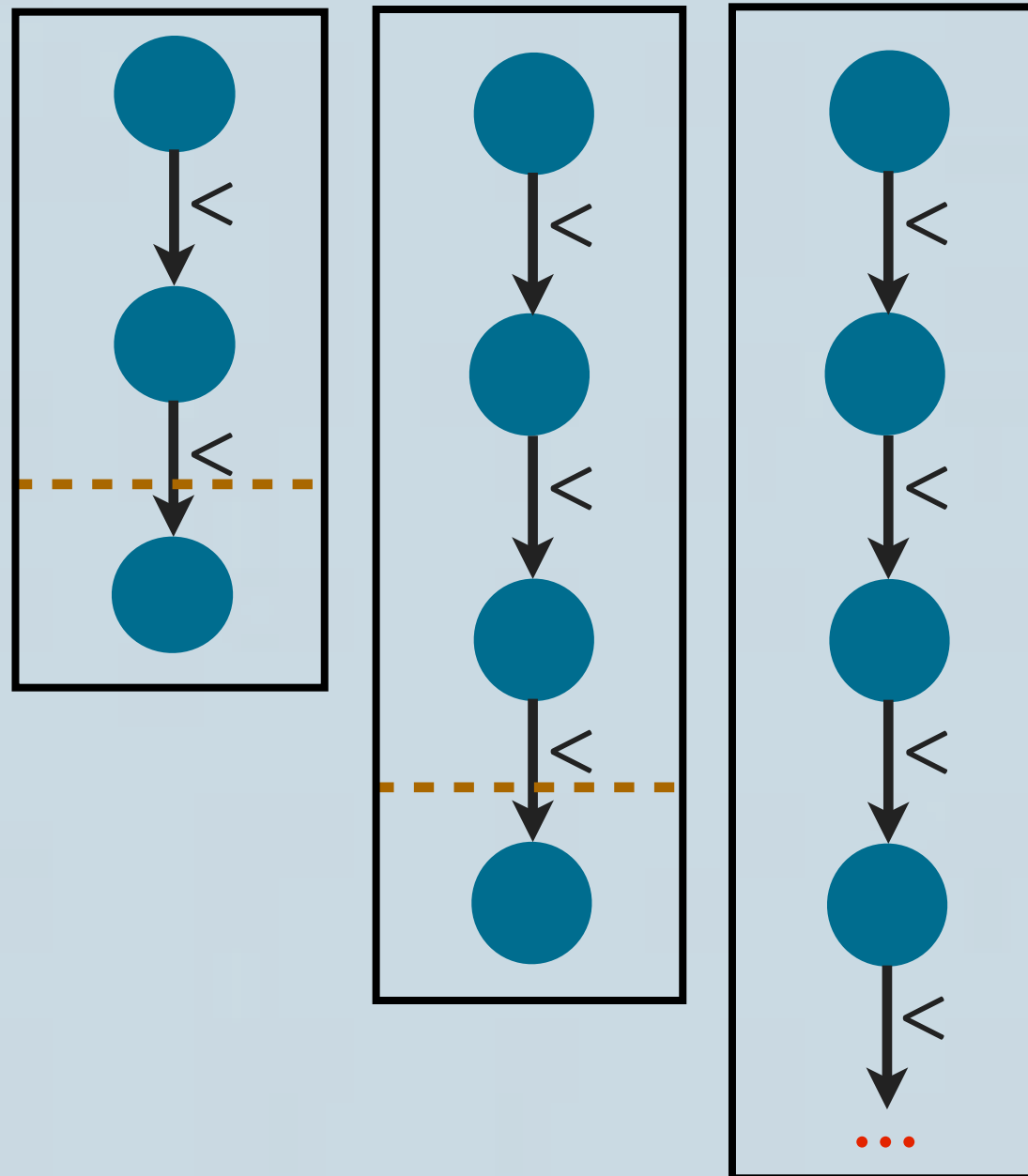
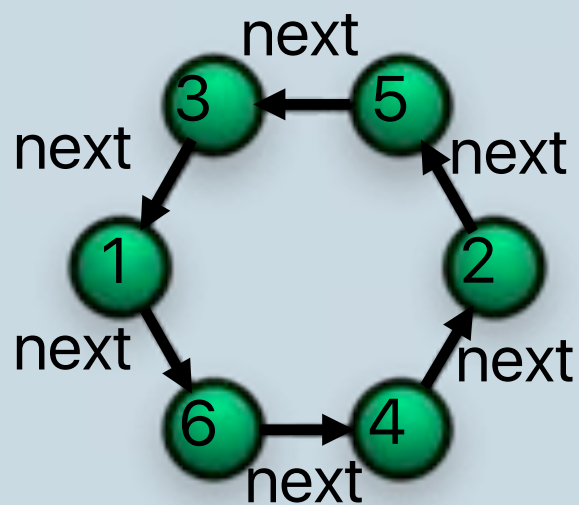
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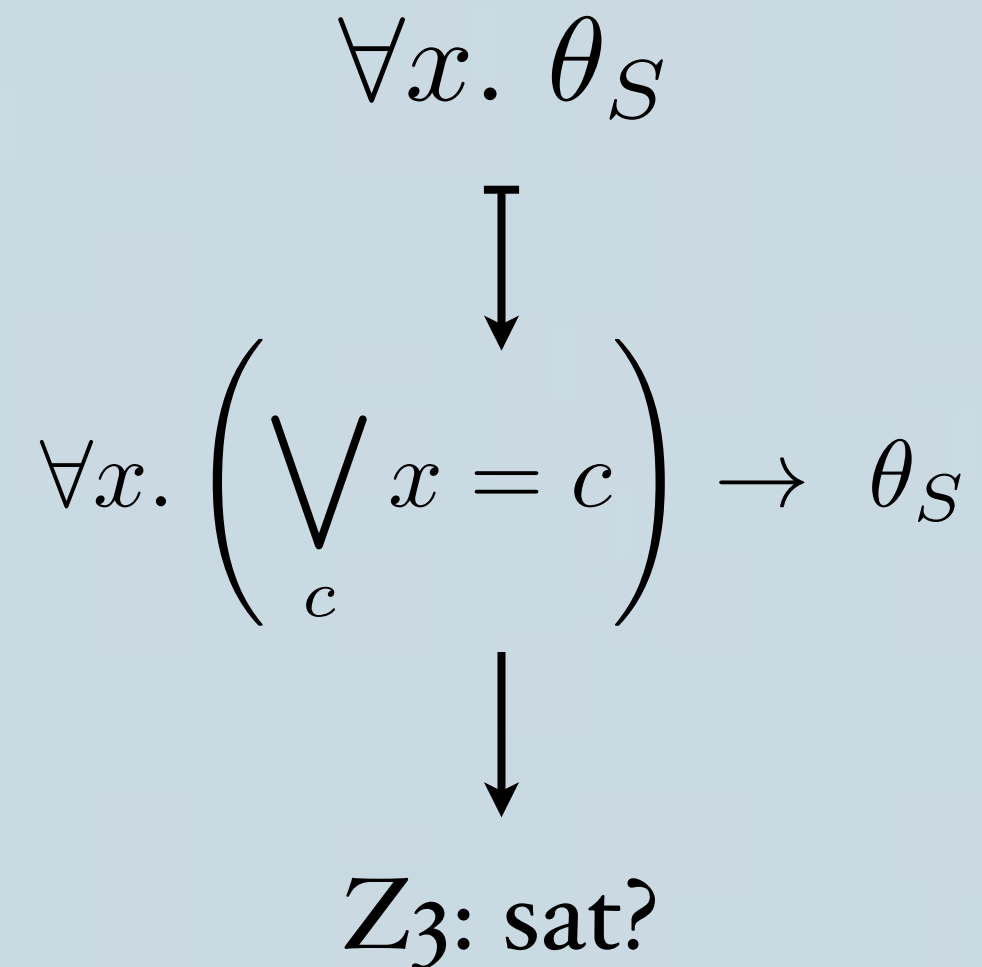
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Implementation of Bound 1



Evaluation

Program	# $\forall$	# f -sk	#Consts	B1 Total	B1 Solve	Baseline Z3
Client-server	14	1	15	58 ms	3 ms	3 ms
List reverse	47	3	15	319 ms	211 ms	50 ms
Learning switch	70	1	7	245 ms	65 ms	33 ms
Hole-punching firewall with ghost	15	1	18	75 ms	4 ms	4 ms
Hole-punching firewall $\exists$ time	32	2	21	102 ms	4 ms	4 ms
Leader-election in a ring (correct)	41	1	21	113 ms	36 ms	27 ms
Leader-election in a ring (incorrect)	40	1	20	1112 ms	1008 ms	—

Summary

Checking $\forall^* \exists^*$ Inductive Invariants

Proof

Cex

sound but
incomplete

powerful as certain
manual reductions
to a decidable class

ability of producing
partial cexs to
facilitate **progress**